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The behaviour of the centre of mass in a ballerina while performing a *Grand Jeté*

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Abstract
In sports, it is very common to see athletes performing jumps, where they impose rotations to their own bodies' elements and intentionaly change their moment of inertia around the centre of mass (CM). When this occurs, weird effects are observed in the body's trajectories. In this work we study a jump called *Grand Jeté* of a ballerina, in which she moves her arms and legs to give the feeling that her body remains 'floating' in the air for a long time. We use a computational model to calculate the position of the ballerina's CM during the jump and to explain quantitatively her motion in terms of a displacement of the CM in the ballerina's frame of reference, which enriches the study of the *Grand Jeté*.

S Supplementary material for this article is available online

1. Introduction

The centre of mass (CM) of a body or a system of bodies is a topic that has been intensively studied by many generations of scientists, especially in mechanics, biomechanics and astronomy contexts [1–9]. Usually, the CM and the centre of gravity of a body coincide, except for some cases where the difference between the two is significant [10].

When objects are tossed as projectiles, neglecting the air resistance, the CM is the only point that always describes a parabolic trajectory [7]. In sports, it is very usual athletes perform jumps where they impose rotations to their own bodies elements (e.g. diving [11] or dancing [12]), or change intentionally their moment of inertia (e.g. skating) [13], around the CM. Although their trajectories are sometimes weird, it is assumed that the CM is located

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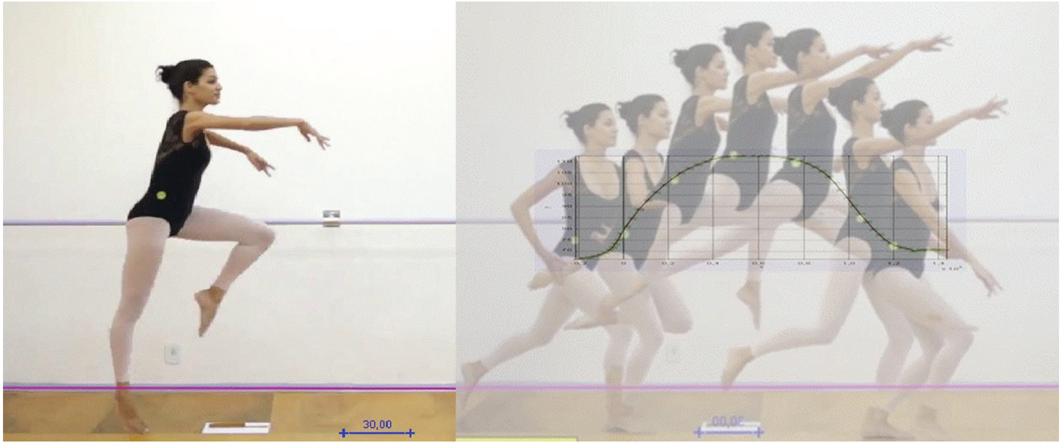

Figure 1. The jump *Grand Jeté* performed by the ballerina reveals a plateau-like step at the top of the trajectory. This weird behaviour is disclosed in the stroboscopic image.

Figure 2. The four steps of the *Grand Jeté*, adapted from the description of Laws [12]: (a) preparing for the jump with one of the legs; (b) rising of the legs and arms; (c) top position of the jump with a split of the legs; (d) landing with the opposite leg.

somewhere in the body or closer the part of the body with the larger mass, without specifying its position [14]. In this work, we study the position of the CM when a ballerina performs a jump called *Grand Jeté*, in which she moves her arms and legs and changes her body's shape, to give the feeling that the body remains 'floating' in the air for a long time.

2. Experimental

The *Grand Jeté* shown in figure 1 is performed by a ballerina that has a green spot bonded to her body, allowing to follow its trajectory. The stroboscopic picture of the jump (right side of figure 1) reveals a weird behaviour as the green spot exhibits a plateau-like step at the top of the trajectory.

Figure 3. Image of the ballerina during the *Grand Jeté*, with colour circles in her body. The location of the estimated mass distribution of the elements in the ballerina's body are represented by red circles. The yellow vertical bar at the centre of the image is the calibrated reference in the video.

The jump can be better described if we consider the four steps adapted from the description of Laws [12]. As shown in figure 2, the wheight of the body is thrown from one foot to the other (steps (a) through (d)). The illusion of the body 'floating' is created as the ballerina changes the body configuration during the jump [12]: the rising of the legs, ideally doing a split at the top of the jump (step (c)), and the arms projected to the front (steps (b) and (c)).

We can have a closer look of the jump by following the trajectory of several points of the body, in order to identify the position of the CM in the body's frame of reference.

Our goal to find the CM of the ballerina begins with a rough estimation of it by considering the standard distribution of mass in a human body, and so we called this point the estimated centre of mass (CMe). According to the literature [15–17], the elements of women's bodies and corresponding percentage of mass distribution are: head and neck (9.4%), upper body (chest, back and abdomen—50.8%), arms (upper arm, forearm and hands—9.6%), thighs (16.6%), and legs (lower leg and feet—13.6%). We call CMe a rough estimation of the CM because the mass distribution may vary from woman to woman which influences the exact position of CM, but this is not a critical factor to the illusion created in the *Grand Jeté*. Figure 3 shows the estimated position of these elements (red circles) in the ballerina's body.

Several colour circles were positioned in the ballerina's upper body, where hypothetically lies the location of the CM [12]. They will be used as reference marks for the interpretation of the 'floating' illusion, as well as for the detailed and quantitative analysis of the position of CM.

The video of the jump *Grand Jeté* was captured by a digital photo camera Canon EOS 5D Mark III with 22.3 MP Full HD resolution at 60 frames per second. For a quantitative characterization of the motion, a calibrated reference bar was included in the video recorded (see supplementary information available at: stacks.iop.org/PhysED/53/025009/mmedia).

3. Results and discussion

As said above, the knowledge of the elements of women's bodies and corresponding distribution of masses allows us to compute the estimated

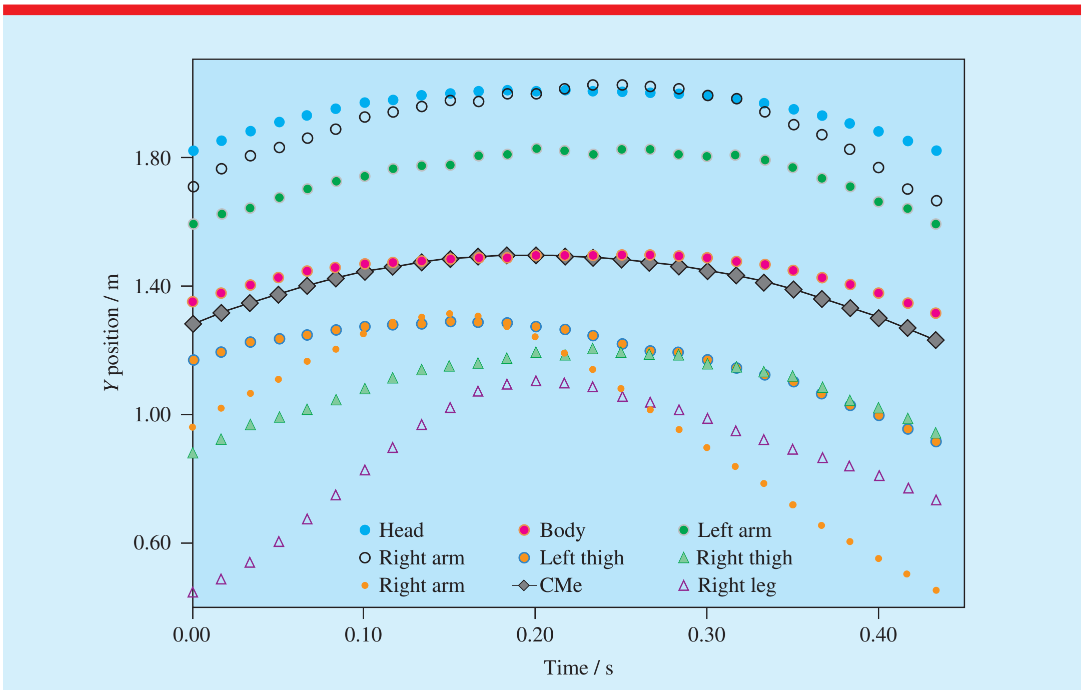

Figure 4. Plot of the Y position described by the estimated mass distribution elements in the ballerina's body. The dark-grey dot-line points represent the estimated CMe computed from these elements.

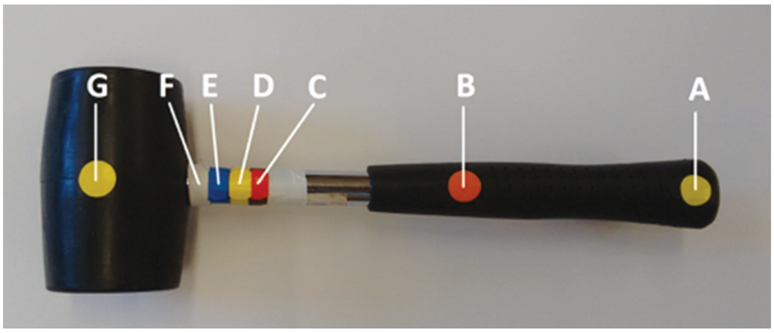

Figure 5. Colour points distributed over a rigid body. Reproduced from [7]. © IOP Publishing Ltd. All rights reserved.

CMe from the percentage of mass distribution of the body's elements during the jump. The vertical position of the estimated mass distribution in the ballerina's body and the corresponding computed CMe along time, are depicted in figure 4. The data were obtained by analysis of the video with Tracker software [18].

It can be easily seen that the head (blue marks) and the upper-body (rose marks) elements present a plateau-like behaviour at the middle part of the jump, giving the so-called 'floating' illusion. The limbs (arms and legs) and the thighs have a significant change of their Y position during the jump, which undoubtedly contribute to the motion of CMe (dark-grey marks) along the upper body's frame of reference.

Two important conclusions are attained from figure 4:

1. The CMe moves up and down along the ballerina's upper-body frame of reference during, respectively, the ascendant and the descendent motion of the arms, legs and thighs (due to the change of the mass distribution).

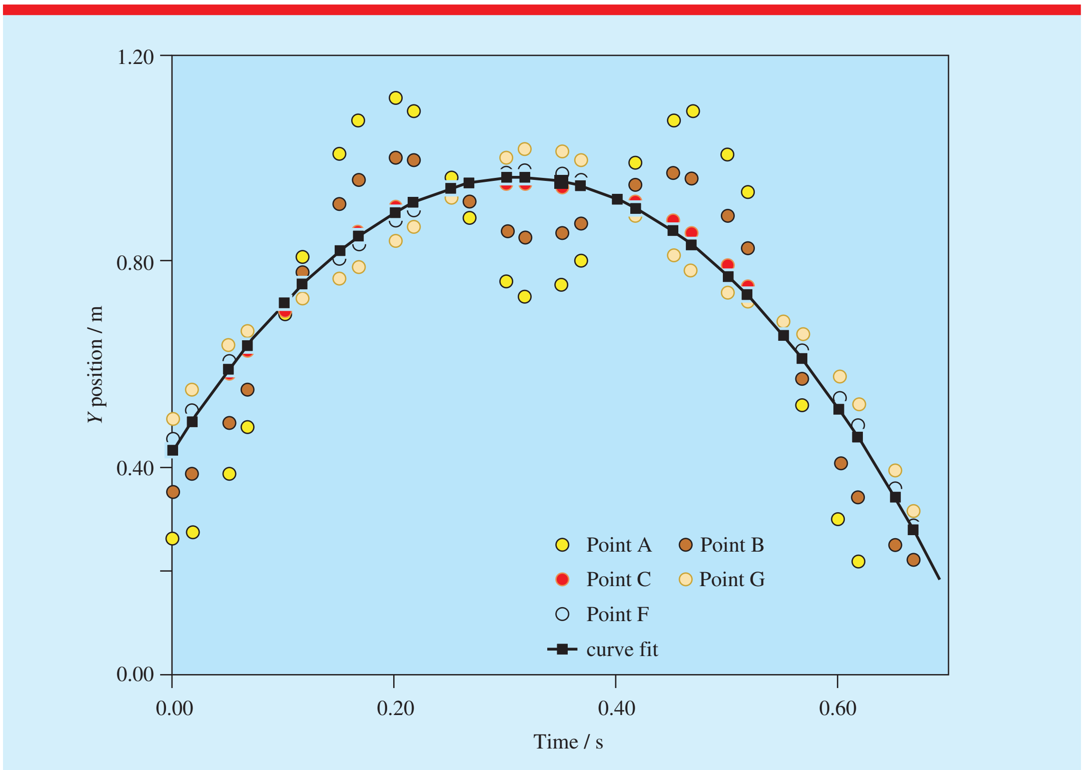

Figure 6. *Y* positions over time for the colour points of figure 5. The data corresponds to the experiment described in [7]. The black dot-line points (curve fit) represent the computed CM from the model.

2. CMe lies very close to the upper body marks, suggesting that the exact location along time of the ballerina's CM is somewhere in the upper-body region, as expected.

The roughly estimated CMe along time confirms that the position of the CM in the body's frame of reference is affected by the motion of the arms and legs during the jump, as predicted by Laws [12]. However, where exactly lies the CM of the ballerina during the jump? How can such knowledge help the ballerina improve her performance and increase the 'floating' illusion in the *Grand Jeté* jump?

Recently, Carvalho and Rodrigues described a computational method for determining the CM, both for bodies where CM is fixed or changes in relation to the body's frame of [19]. We intend to use this model to find the location of the CM along time during the *Grand Jeté*, in the expected area (the ballerina's upper-body region).

To illustrate and test how the model works, we decided to use data from a previous test experiment, where several colour points of a rigid body (figure 5) were followed by video recording. The position of the CM was fairly well identified [7] as the blue mark (point E), whose vertical representation as a function of time was a parabola.

For this test analysis, all sets of data were considered except the one corresponding to the CM (point E). Figure 6 shows that the *Y* position over time for the distinct colour points are quite different from each other, but nevertheless the parabolic fit (black dot-line points) obtained with the model matches quite well the blue marks (point E), corresponding to the CM as confirmed in figure 7.

So, despite the complex data provided by the weird trajectory of the points, we confirm that the model can find the fixed CM of the hammer (rigid body) as predicted by the authors [7]. In the ballerina's case, the computed CMe suggests that the most probable location of CM in the *Grand Jeté* along time, is somewhere in the upper-body element (figure 4), which in a reasonable

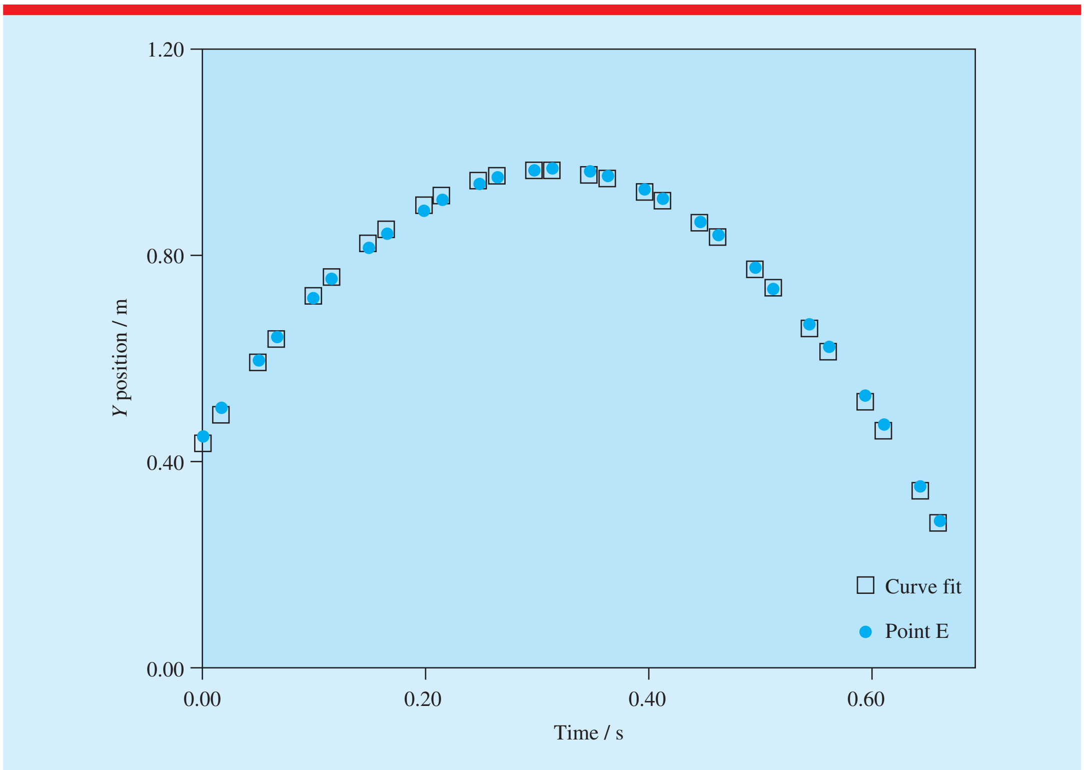

Figure 7. *Y* positions over time of point E of figure 5 and the computed CM (curve fit) from the model. The match is very good.

approximation can be considered as a rigid body-like region (i.e. the relative positions of the external coloured marks are constant). Therefore, we can apply the fitting model of [19] in the way described for the hammer, to compute a more rigorous location of the ballerina's CM. Note that unlike the hammer, the CM of the ballerina changes along the upper-body region, although that is not a problem for the model (see [19]).

Figure 8 shows a plot of the *Y* positions of the colour circles in the ballerina's body, as well as the CM computed from the model. The parabolic shape of CM in the body's frame of reference indicates a similar behaviour to that found for the estimated CMe in figure 4.

Analysing figures 4 and 8 we can see that in the ascending phase of the jump, the legs and arms are lifted and contribute to the rise of CM across the upper-body's frame of reference. Consequently, the head and the upper-body paths are different from the parabolic trajectory of the CM relatively to the floor.

At the middle of the jump, the upwards displacement of CM is essentially due to the split of the legs, while the ballerina maintains her head and upper-body in the air in a nearly horizontal stable trajectory (the plateau-like region). To extend this effect during the descendent phase of CM, she starts to close her legs to keep her head and upper-body at nearly the same height from the ground. This plateau-like region lasts for about 0.20 s. From a spectator's point of view that gives the so-called illusion of 'floating'. The best performance is achieved if the total split occurs when the CM is at the maximum height.

So, the 'floating' illusion will therefore depend on the synchronization between the vertical position of the upper-body and the speed with which the limbs (legs and arms) are relocated, to increase the plateau-like effect as much longer as possible. This knowledge helps the ballerina to get the best timing to reposition the parts of her body and improve her technique and performance of the *Grand Jeté* jump.

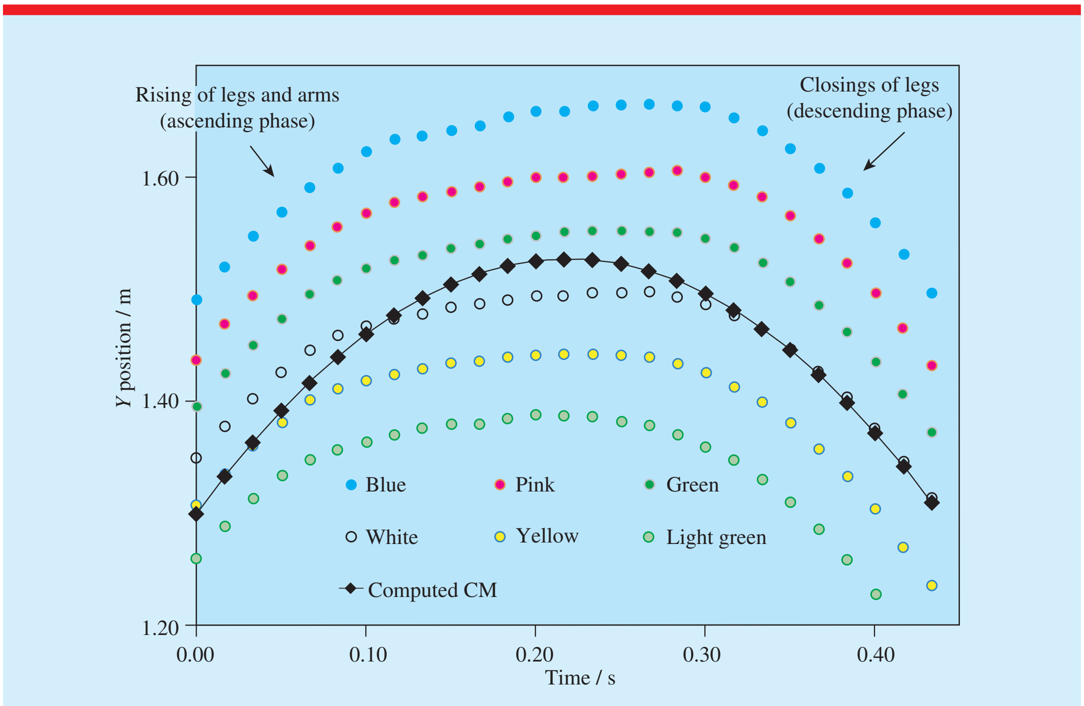

Figure 8. Plot of the Y positions described by the colour circles in the ballerina's body. The black dot-line points represent the computed CM, which is described by the parabolic function $Y = -4.7\,t^2 + 2.1\,t + 1.3$ (m). The computed acceleration is of about 9.4 m s^{-2}.

4. Conclusions

In this work, we have confirmed that a ballerina changes the position of her body's CM in her own upper-body frame of reference by moving her legs and arms during a jump, as suggested by Laws in his qualitative approach to the *Grand Jeté* jump. This is a counter-intuitive result that is never observed for strictly rigid bodies.

The synchronization between the vertical position of the body and the speed of the limbs creates the weird plateau-like trajectory of the head and upper-body elements, giving the illusion of a ballerina 'floating' in the air. The timing these gestures are executed contribute to the performance of the ballerina, therefore quantitative knowledge of how these movements influence the duration of the 'floating' illusion takes an important role in the ballerina's technique.

To enrich the study of the *Grand Jeté* we used a fitting model for computing the parabolic motion of the ballerina's CM, thus providing a more rigorous location of the CM with time. The good performance of the fitting in explaining quantitatively the *Grand Jeté* jump recommends that the model may be useful to athletes and coaches for improving the technique and increasing the 'floating' illusion in the jump.

Is the ballerina's 'floating' a real effect or just an illusion? Physically and considering the model of the CM, the 'floating' is an illusion created by the way the CM moves along the ballerina's upper-body frame of reference. Curiously, the dancers do not realize it while they are jumping. Artistically, it is a real optical effect produced by the complex trajectories of the ballerina's body elements. Spectators are delighted when they watch the head follow a path almost completely flat at the highest point of the trajectory as if it was 'floating' over an imaginary line. The better the effect, the greater the emotions created by the jump, and that is the real 'magic' of ballet.

The video analysis and the experimental computation shown in this paper can be done in other sports to study and understand the motions of athletes, and eventually to optimize their performances. Results of these studies will be published elsewhere.

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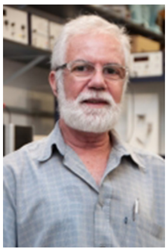

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