Demonstrations of magnetic phenomena: measuring the air permeability using tablets

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Abstract
We use a tablet to experimentally determine the dependencies of the magnetic field (B) on the electrical current and the axial distance from a coil (z). Our data shows good precision on the inverse cubic dependence of the magnetic field on the axial distance, $B \propto z^{-3}$. We obtain the value of air permeability μ_{air} with good accuracy. We also observe the same dependence of B on z when considering a magnet instead of a coil. Although our estimates are obtained through simple data fits, we also perform a more sophisticated error analysis, confirming the result for μ_{air}.

The use of tablets and smartphones in science education expands the possibilities of approaches that motivate students to better understand several physical phenomena [1–4]. In particular, tablets have been shown to be good tools for measuring the magnetostatic responses in current-carrying wires. This interesting article looks at magnetic field sensoring [1]. It reports a simple way of experimentally obtaining the linear dependence between the magnetic field B and the number of turns N in a current-carrying coil using an app for a tablet such as an iPad [5]. However, the additional dependencies of B are still not discussed, some of which we will show in this paper, leading to a wider description of this kind of system.

We determine the dependencies of B on the electric current I and the axial distance z in a coil in suitable conditions using a tablet and the same MagnetMeter app [5] (and suggest a similar Android app [6]). We also perform similar experiments with a small magnet instead of a coil. For the coil, we also make a good estimate of the magnetic permeability of the air $\mu_{\mathrm{air}} \cong \mu_0 \equiv 4\pi \times 10^{-7}$ Hm^{-1}.

The demonstration set we used is composed of an electrical circuit, a ruler and a book (see figure 1). The circuit is formed by the following components: a wire-wound potentiometer with resistance up to 30 Ω; a resistor with 10 Ω; an electrical source from a cell phone (max. output current ~0.9 A, dp = 3.7 V); a digital multimeter and a coil (internal diameter $2R_i$ = 1.910 cm and external diameter $2R_e$ = 2.420 cm and N = 62 turns).

Next, we describe the circuit assembly, which is relatively easy to build. This circuit is formed in such a way that the potentiometer

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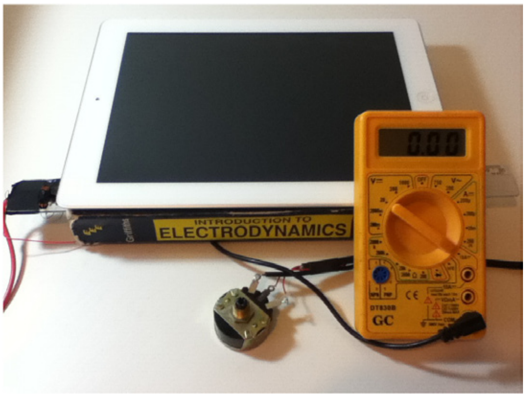

Figure 1. A photograph of the experimental setup.

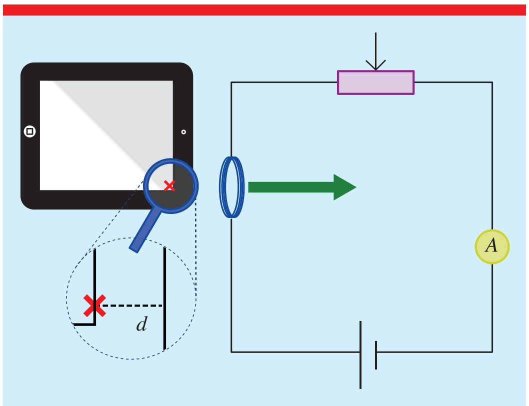

Figure 2. Schematic representation of the coil experiment. The red cross indicates the position of the magnetometer inside the tablet, while the green arrow corresponds to the *z* axial displacement of the coil.

enables the variation of the current in the coil, which is measured by the ammeter. However, for safety issues, it could also be necessary to add the extra resistance of 10 Ω to avoid high currents. The potentiometer has three terminals. The middle one has to be connected to the coil, while each of the other terminals are connected to the resistor and to the negative source terminal. There is no need to worry about the connection order of these two terminals because the circuit should behave as expected in both ways, although one should be careful about the direction in which the potentiometer will increase the current value measured by the ammeter. In order to simplify the schematic representation, we replaced the potentiometer details with a current source symbol, as can be seen in figure 2.

As a standard procedure throughout, before starting the experiment we pressed the red button on the MagnetMeter app to set any other relevant magnetic interference, such as the Earth's magnetic field, aside. In the first experiment, we pulled the coil up close to the upper right edge of the tablet (see figure 2). We fixed the axial distance between the coil and the magnetometer at $z = 4.8$ cm. It is crucial that one takes into account the distance d relative to the localization of the magnetic sensor inside the tablet, adding it to the value measured by the ruler (for the tablet, we use $d \sim 1.8$ cm).[5] Next, we increased the current

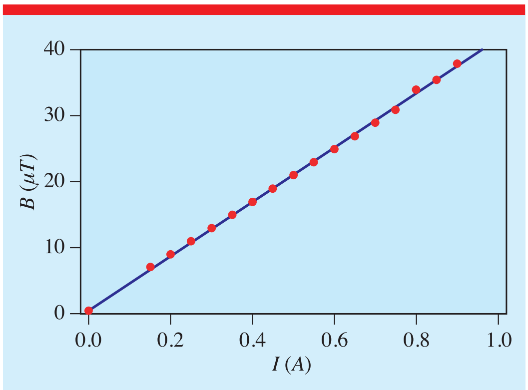

Figure 3. The magnetic field linear dependence on the electric current for the coil. The blue line represents the linear fit and the red points correspond to the obtained data. The values of *a* and *b* vary according to the coil radii and the axial distance from the magnetic sensor.

by equal amounts $\delta I = 0.05$ A, writing down the magnetic fields measured by each corresponding current, as plotted in figure 3. The data adjust was realized using the 'fit' command from *Gnuplot* [8], with

$$\begin{aligned} B(I) &= aI + b, \\ a &= (41.0247 \pm 0.2571)\mu T/A \quad \text{and} \\ b &= (0.611347 \pm 0.143)\mu T. \end{aligned} \tag{1}$$

The second experiment consists of an analysis of the magnetic field dependence on *z* for both the coil and the magnet [7]. We started by holding the ruler between the book pages and positioning

[5] To determine the inner distance *d*, we use a magnetized needle that is detected by the magnetometer. Moving the needle tip on the screen allows us to detect the location of the maximum magnetic field, i.e. the position of the magnetometer inside the tablet or phone.

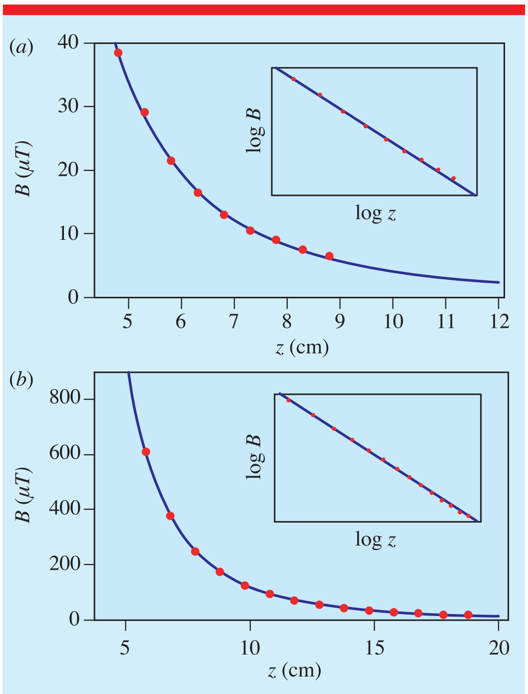

Figure 4. The magnetic field versus distance for the (a) coil and (b) magnet. The inset shows the same data on log–log scale.

Table 1. Magnetic field fitting, $B(z) = a(z/z_0)^b z_0^{-3}$ for the coil and magnet.

Parameters	$a \pm \delta a$ [cm$^3\mu$T]	$b \pm \delta b$
Coil	4624.6 ± 307.4	−3.05112 ± 0.03917
Magnet	142012 ± 2903	−3.09232 ± 0.01217

the tablet above the book, with its magnetic sensor facing the coil or the magnet. For the coil we increased the current up to its maximum $I \sim 0.9$ A, which is not necessary in the case of the magnet due to its permanent magnetization. We subsequently moved the coil (magnet) by equal displacements in the green arrow direction as indicated in figure 2, and took note of the magnetic field showed by the app for each distance. In figure 4 we plotted the experimental data of B as a function of z obtained by the demonstration set for (a) the coil (see table 2) and (b) the magnet, respectively. In addition, we performed a data fit using $B(z) = a(z/z_0)^b z_0^{-3}$, where z_0 = 1 cm has been included in order to keep a standard unit for the parameter a[cm$^3\mu$T], with the b parameter being dimensionless. We show in table 1 the

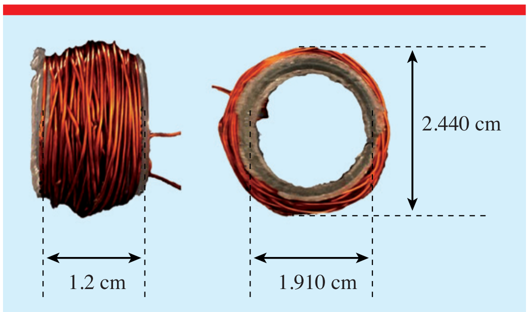

Figure 5. The spatial dimensions of the coil used in this work. Notice that the coil width is 1.2 cm, and we use its half width as the referential point to measure the distance z from the magnetic field sensor. The inner and outer diameter are 1.910 and 2.440 cm, respectively.

Table 2. The magnetic field $B(z)$ and z for the coil.

Magnetic field (B) [μT]	Axial distance (z) [cm]
38.5	4.8
29	5.3
21.5	5.8
16.5	6.3
13	6.8
10.5	7.3
9	7.8
7.5	8.3
6.5	8.8

corresponding values obtained in the data fitting. For both cases we have an excellent agreement with the expected z^{-3} dependence [9].

We present in figure 5 the dimensions of the coil used. From figure 5 one finds $R_M = (1.910 + 2.440)/4$ cm = 1.088 cm, where R_M is the mean radius of the coil. We also know the electrical current value and the number of turns of the coil. This information allows us to obtain an estimate of the magnetic permeability $\mu_{air} \cong \mu_0$. The inverse cubic dependence of the magnetic field of the coil is consistent with the magnetic field generated by a pure magnetic dipole (m) in its axis [9], given by

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{m}{z^3}\hat{z}, \qquad (2)$$

where $m = NI\pi R_M^2$, N = 62 turns, I = 0.9 A and R_M is the mean radius of our coil. Therefore, we impose $b = -3$ for our data and we leave a' as the only parameter in the data fit. The data fit performed for the points displayed in figure 4

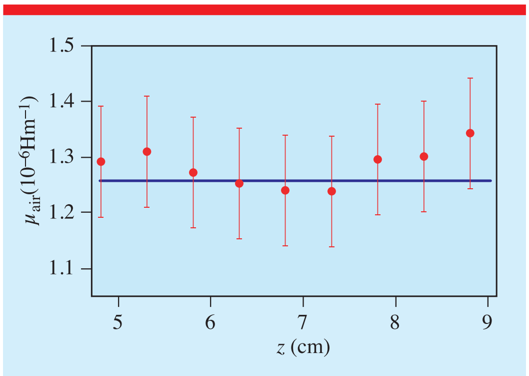

Figure 6. Air permeability values obtained from the experimental data evaluated at different axial distances (red points). The blue line indicates the established value for μ_{air}.

returns $a' = 4240 \pm 25.81$, with a standard deviation of less than 1% . The relation between the a' coefficient for the coil and μ_{air} is given by

$$\mu_{air} \cong \frac{a'2\pi}{m} = \frac{2a'}{NIR_M^2}. \quad (3)$$

Converting all the units to their SI values, the result leads to $\mu_{air} \cong 1.298 \times 10^{-6}$ Hm^{-1}, in good agreement with the expected value of $\mu_0 \equiv 4\pi \times 10^{-7} \approx 1.2566370614 \times 10^{-6}$ Hm^{-1}.

Although this estimation for μ_{air} is already good enough, we also perform another procedure to evaluate the air permeability and the error analysis.

We use the coil data points from table 2, and replace these values in equation (2) to find the value of μ_{air} for each single point. We also consider the following uncertainties in the experimental measurements: $\delta B = 0.5\ \mu$T, $\delta I = 0.01$ A, $\delta z = 0.001$ m, $\delta N = 1$ and $\delta R_M = 0.00005$ m. There is a precision difference between R_M and z because we used different measurement devices. For z we use a simple ruler, and for R_M we use a calliper rule. For the error calculation we use the following variance formula, taking into account all the independent variables [10]:

$$\sigma_{\mu_{air}} = \sqrt{\left(\frac{\partial \mu_{air}}{\partial B}\right)^2 \delta B^2 + \ldots + \left(\frac{\partial \mu_{air}}{\partial R}\right)^2 \delta R^2}. \quad (4)$$

Each partial derivative of the previous expression is evaluated at the average values of the magnetic field and of axial distances, in such a way that we obtain the same uncertainty value for all the points, as a mean value. The uncertainty $\sigma_{\mu_{air}} = 0.1 \times 10^{-6}$ Hm^{-1} tells us that the experiment enables the evaluation of μ_{air} with two significant figures. We show in figure 6 the μ_{air} values obtained for different axial distances, given by red dots with corresponding uncertainties. The expected value of μ_{air} is also exhibited in the figure given by the blue line. Notice the relatively small data deviations from the expected value, which means that following this procedure, we also obtained a fair estimate for μ_{air}.

Unfortunately, in this experimental analysis it was not possible to determine the value of μ_{air} using the magnet. In fact, all that one is able to make is an estimate of the magnet magnetic dipole m, assuming the value for μ_{air}.

From the circuit made, we obtained a linear dependence between B and I. In addition, we observed the same proportionality $B \propto z^{-3}$ for both the coil and the magnet, enabling a discussion of the parallel between them. Finally, we also made a fair and simple estimate of the magnetic permeability μ_{air}, even under the limitations of our experimental device. Unfortunately, our analysis applied to the magnet does not provide the value of μ_{air}. In the magnet case, all one may obtain, assuming the value for μ_{air}, is an estimate of its magnetic dipole m. For further experiments, we suggest a study of the dependence of the magnet on distance for other geometries, such as a long straight wire or a current on a plane sheet of steel.

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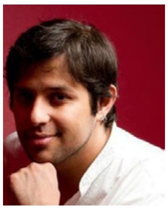

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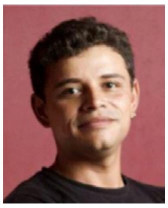

Leonardo Vieira received his masters degree in physics education from Universidade Federal do Rio de Janeiro in Brazil in 2013. His research field is focused on the use of smartphones and tablets as detection instruments in experiments in physics. He has been teaching at high school level since 2009.