The Physics Teacher

The Goldfish Over the Rainbow

Carlos E. Aguiar, Antonio C. F. Santos, Eric B. Lopes, and Walter S. Santos

Citation: Phys. Teach. **51**, 522 (2013); doi: 10.1119/1.4830061
View online: http://dx.doi.org/10.1119/1.4830061
View Table of Contents: http://tpt.aapt.org/resource/1/PHTEAH/v51/i9
Published by the American Association of Physics Teachers

Related Articles

OOMPH
Phys. Teach. 51, 518 (2013)

Correction to “Galilean moons, Kepler's third law, and the mass of Jupiter,” Phys. Teach. 51, 428 (Oct. 2013).
Phys. Teach. 51, 517 (2013)

Layton and Rodriguez respond
Phys. Teach. 51, 516 (2013)

• U.S. National Science Board releases Science and Engineering Indicators 2012
Phys. Teach. 51, 573 (2013)

• U.S. National Science Board releases “Preparing the Next Generation of STEM Innovators: Identifying and Developing Our Nation's Human Capital”
Phys. Teach. 51, 573 (2013)

Additional information on Phys. Teach.

Journal Homepage: http://tpt.aapt.org/
Journal Information: http://tpt.aapt.org/about/about_the_journal
Top downloads: http://tpt.aapt.org/most_downloaded
Information for Authors: http://www.aapt.org/publications/tptauthors.cfm

The Goldfish Over the Rainbow

Carlos E. Aguiar and Antonio C. F. Santos, Universidade Federal do Rio de Janeiro, RJ, Brasil
Eric B. Lopes, Instituto Federal Fluminense, RJ, Brasil
Walter S. Santos, Colégio Pedro II, RJ, Brasil

A goldfish isn't always visible inside its fishbowl. If the fish gets sufficiently close to the bowl glass, it will disappear for certain angles of observation. In a recent paper in this journal,[1] Zhu and Shi explained the effect in terms of total internal reflection. We show in what follows that the phenomenon involves some richer optics: a rainbow defines one of the boundaries of the "blind zone" where the fish disappears.

Total internal reflection

Let us consider a spherical fishbowl of radius R, with the goldfish at distance a from its center. Figure 1 shows a light ray starting at the fish (the red dot) and leaving the bowl after refraction. The ray initially makes an angle γ with the line joining the fish to the center of the bowl, and reaches the wall with angle of incidence i.

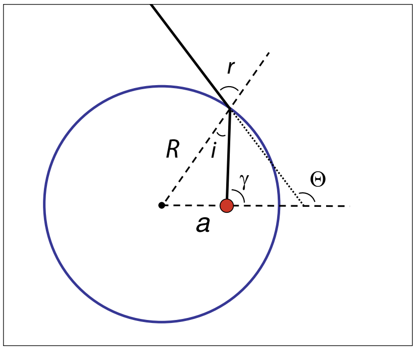

Fig. 1. Light ray coming from the fish (red dot) in the bowl.

The angle of incidence is related to γ by the law of sines

$$\sin i = x \sin \gamma, \tag{1}$$

where $x = a/R$ is the distance from the center in units of the bowl radius. After refraction at the surface, the light ray leaves the fishbowl making an angle r with the direction of normal incidence (see Fig. 1). Using Snell's law and Eq. (1) we find that

$$\sin r = nx \sin \gamma, \tag{2}$$

where n is the refraction index of water relative to air (we neglect the width of the glass wall).

The role of total internal reflection on the disappearance of the goldfish is understood by noting that, since $\sin r \leq 1$, Eq. (2) implies that light rays are refracted out of the bowl only for angles such that

$$\sin \gamma \leq 1/(nx). \tag{3}$$

If the goldfish is not too far away from the center of the bowl—more precisely, if $a < R/n$—then $nx < 1$ and Eq. (3) poses no restriction on the angle γ. In this case every light ray coming from the fish leaves the bowl. On the other hand, Eq. (3) defines an angular range

$$\pi/2 - \delta < \gamma < \pi/2 + \delta, \tag{4}$$

in which total internal reflection prevents the light rays from escaping the bowl. The width of this range is 2δ, where $\delta = \cos^{-1}(1/nx)$, a result following from Eq. (3) and the trigonometric relation $\sin \gamma = \cos(\gamma - \pi/2)$.

It is useful to define the "observation angle" Θ between the outgoing ray and the line passing through the goldfish and the center of the bowl. From Fig. 1 we see that

$$\Theta = \gamma + r - i. \tag{5}$$

Should Θ increase monotonically with γ, the angular range of total internal reflection given by Eq. (4) would transform into the interval

$$\pi - i_c - \delta < \Theta < \pi - i_c + \delta, \tag{6}$$

where $i_c = \sin^{-1}(1/n)$ is the critical angle for reflection and we have used that the corresponding refraction angle is $\pi/2$. According to Ref. 1 the fish should be invisible from these observation angles. However, as we discuss next, the "blind zone" is somewhat smaller than the range defined in Eq. (6), and this has to do with the appearance of a rainbow.

A rainbow at the fishbowl

Light rays originating at the goldfish and leaving the fishbowl are shown in Fig. 2. The fish is represented by the red dot to the right of the center of the bowl. For clarity we have not drawn the rays inside the bowl. The blind zone from which the fish cannot be seen is the large gap between the rays. To the right of the figure, the blind zone is limited by rays leaving the bowl tangentially. This corresponds to the onset of total internal reflection, given by the lower limit in Eq. (6). The other boundary of the blind zone is more complicated. Close inspection of Fig. 2 shows that the rays defining this boundary are not tangential to the bowl, indicating that

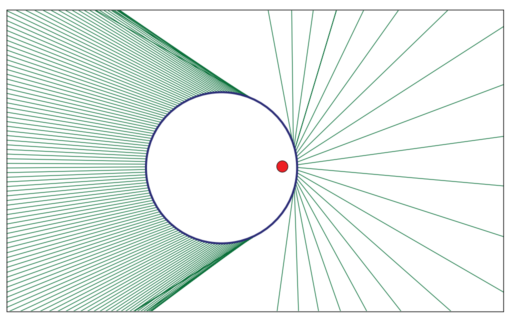

Fig. 2. Light rays coming from the fish (red dot on the right) as they emerge from the fishbowl. The figure is an implementation of the refraction law, Eq. (2), in a dynamic geometry environment.[2,3]

 DOI: 10.1119/1.4830061

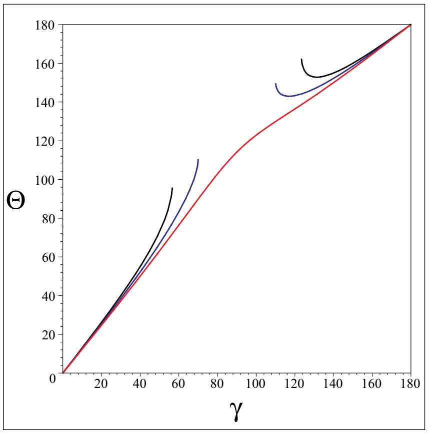

Fig. 3. Observation angle Θ as function of γ for three values of the distance parameter: x = 0.7 (red curve, bottom), 0.8 (blue curve, middle), and 0.9 (black curve, top).

internal reflection is not the mechanism responsible for the disappearance of the fish on this side. What happens in this region is better seen in Fig. 3, which shows plots of the observation angle Θ as function of γ [this function is easily calculated putting together Eqs. (1), (2) and (5)]. Three curves, corresponding to different values of the distance parameter x, are shown. For $x = 0.7$ the fish is always visible, as in this case $nx < 1$ (we took $n = 1.33$ for the water-air refractive index). For the larger values of x (0.8 and 0.9), $nx > 1$ and a gap appears in the curves, corresponding to the invisibility zone. The γ range of the gap is given by Eq. (4), but the observation angle gap is shorter than the one in Eq. (6) due to the non-monotonic behavior of near the upper boundary—it first decreases to values smaller than $\pi - i_c + \delta$ [the upper limit in Eq. (6)], then goes through a minimum and finally increases steadily.

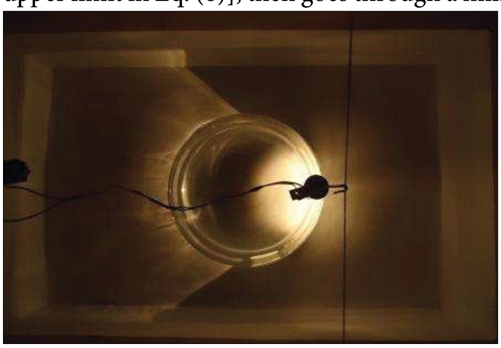

Fig. 4. Light from a lamp in a glass container filled with water. The lamp (or a fish) cannot be seen from the dark zone. The bright boundary between the dark and illuminated areas is a rainbow.

Fig. 5. A closer view of the rainbow, showing the color separation due to dispersion.

Besides shortening the blind zone, the minimum of $\Theta(\gamma)$ has another important consequence. An extremal point (a maximum or minimum) in $\Theta(\gamma)$ is the telltale sign of a rainbow at the corresponding observation angle.[4,5] This means that the blind zone ends at a rainbow. The clustering of emerging rays at the rainbow angle makes this endpoint appear very bright and sharply defined. At the other boundary there is a gradual transition from light to dark, associated with the onset of total internal reflection.

The existence of this rainbow is confirmed by Fig. 4. It shows the photo of a circular container filled with water, with a small incandescent lamp placed inside (compare it to the simulation in Fig. 2). The illuminated areas outside the container are the points from where one can see the lamp (or goldfish), and the darker regions correspond to the blind zone. The rainbow is easily noted at the sharp boundary between the illuminated and dark zones. The somewhat diffuse limit at the other side of the blind zone is associated with total internal reflection. A closer view of the rainbow is shown in Fig. 5 and reveals the expected color separation due to dispersion.

In summary, we have shown that a goldfish becomes invisible in its fishbowl not only because of total internal reflection, but also because of a rainbow. Depending on where we are looking from, we only see the fish after we cross the rainbow.

References

1. Zhu Yuhua and Shi Fengliang, "Why does the goldfish disappear in the fishbowl?" *Phys. Teach.* **47**, 424–426 (Oct. 2009).
2. Tabulae, dynamic geometry software, http://tabulae.net.
3. C. E. Aguiar, "Óptica e geometria dinâmica," *Revista Brasileira de Ensino de Física* **31**, 3302 (2009).
4. R. J. Whitaker, "Physics of the rainbow," *Phys. Teach.* **12**, 283–286 (May 1974).
5. J. D. Walker, "Multiple rainbows from single drops of water and other liquids," *Am. J. Phys.* **44**, 421–433 (May 1976).

Carlos E. Aguiar, Instituto de Física, Universidade Federal do Rio de Janeiro, C.P. 68528, Rio de Janeiro, 21941-972, RJ, Brasil; carlos@if.ufrj.br