Relatividade: do Teorema de Pitágoras às ondas gravitacionais *na velocidade da luz*

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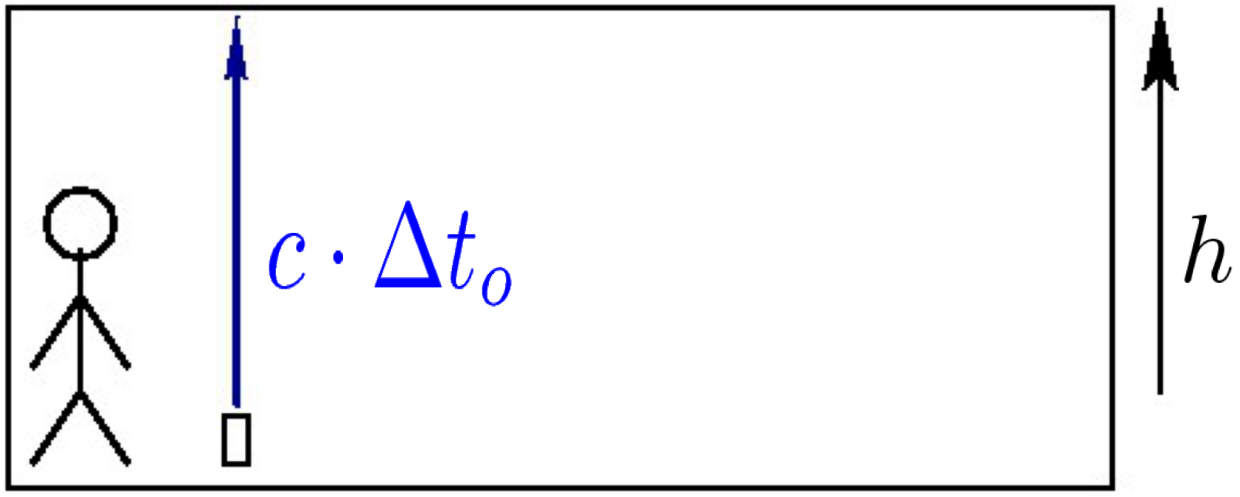
maio de 2017

Método Científico

- Suposição
- Previsão
- Teste Experimental
- Conclusão

$$c = \text{constante}$$

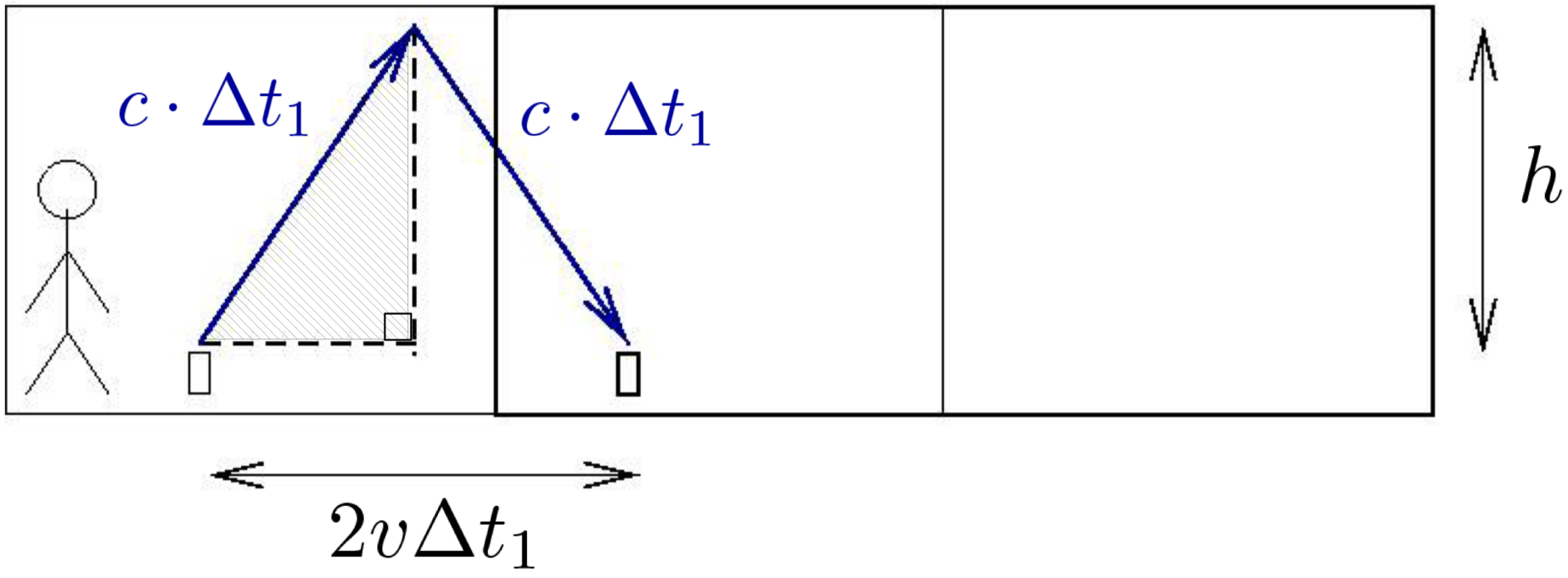
$$\boxed{v = 0}$$



IDA (OU VOLTA):

$$h = c \cdot \Delta t_o$$

$$v \neq 0$$



$$(c \cdot \Delta t_1)^2 = h^2 + (v \cdot \Delta t_1)^2$$

$$(\Delta t_1)^2(c^2 - v^2) = h^2 = (c \cdot \Delta t_o)^2$$

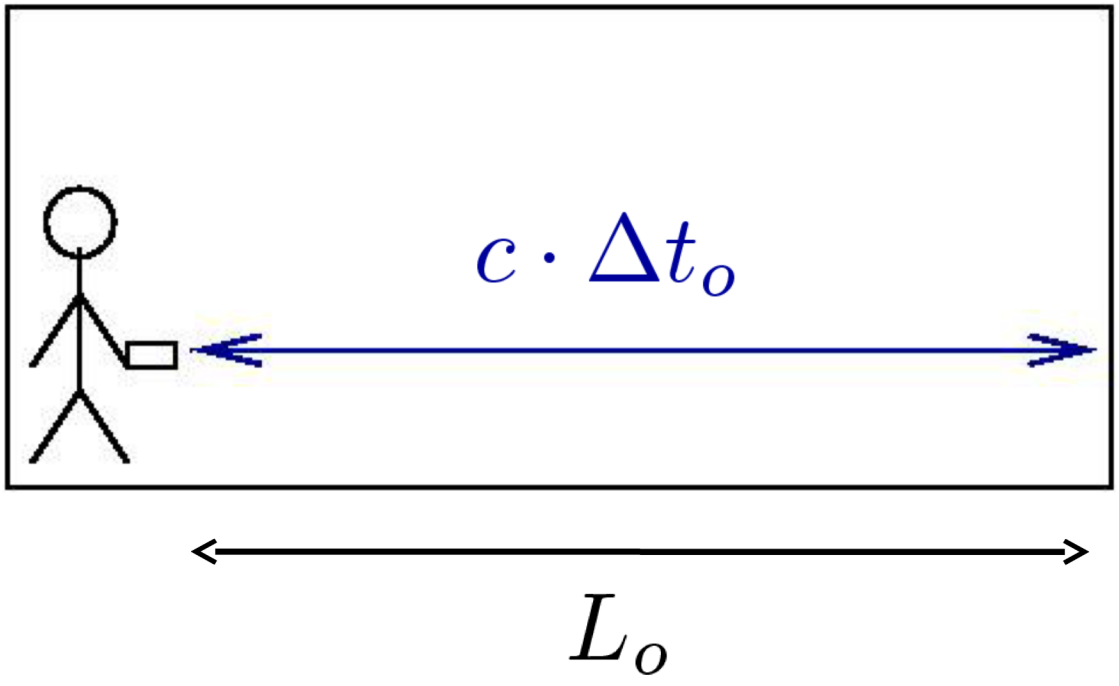
$$\Delta t_1 = \frac{c}{\sqrt{c^2 - v^2}} \Delta t_o$$

$$\Delta t_1 = \frac{1}{\sqrt{1 - v^2/c^2}} \Delta t_o \quad *$$

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \gamma > 1$$

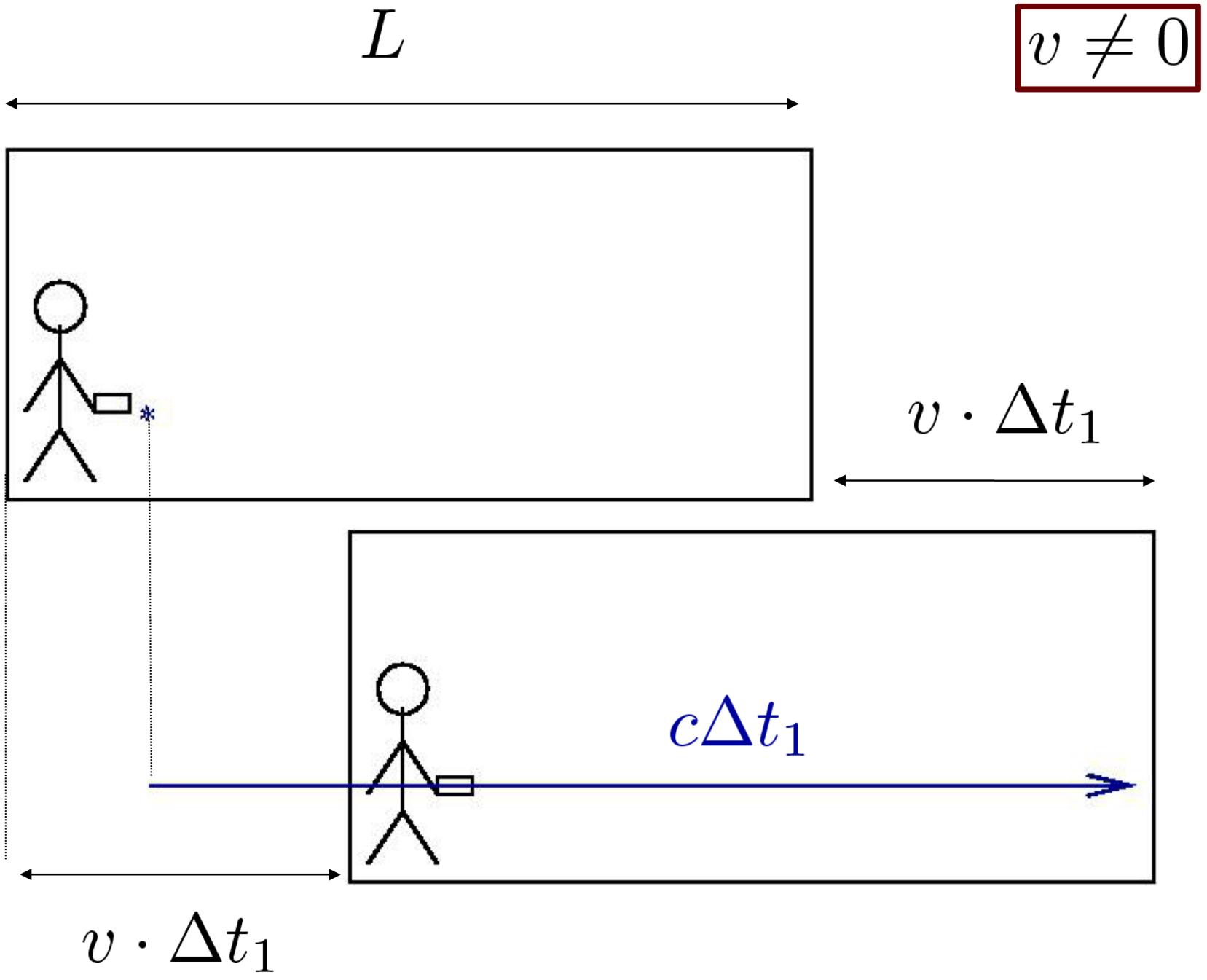
$$\Delta t_1 > \Delta t_o$$

$$v = 0$$



IDA + VOLTA:

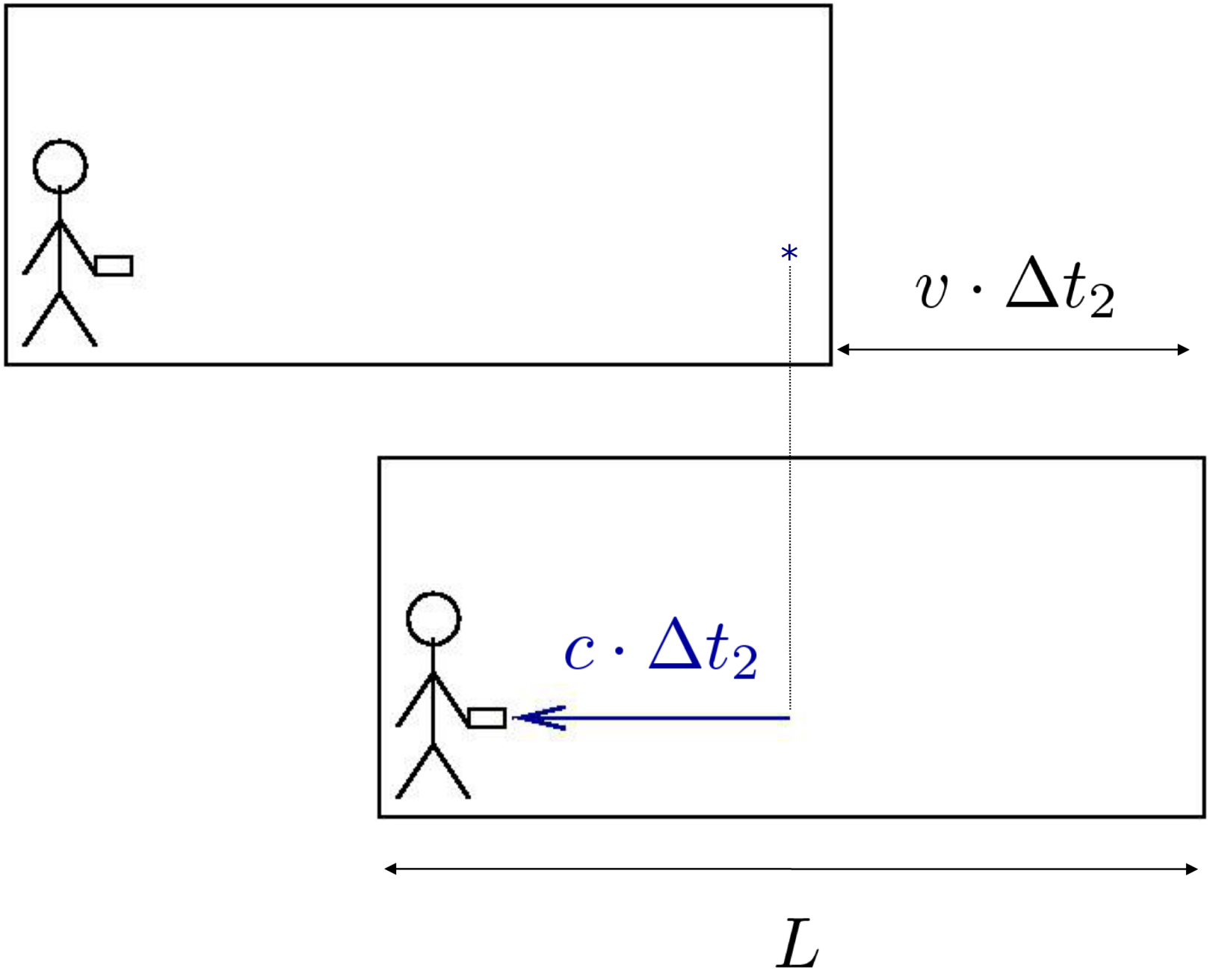
$$2L_o = c \cdot \Delta t_o \quad \Rightarrow \quad \Delta t_o = \frac{2L_o}{c}$$


L
$v \neq 0$
$v \cdot \Delta t_1$
$c\Delta t_1$
$v \cdot \Delta t_1$

$$L + v \cdot \Delta t_1 = c \cdot \Delta t_1$$

$$L = (c - v)\Delta t_1$$

$$\boxed{\Delta t_1 = \frac{L}{c - v}}$$


$v \cdot \Delta t_2$
$c \cdot \Delta t_2$
L

$$L - v \cdot \Delta t_2 = c \cdot \Delta t_2$$

$$L = (c + v)\Delta t_2$$

$$\boxed{\Delta t_2 = \frac{L}{c + v}}$$

$$\boxed{\Delta t_1 = \frac{L}{c - v}}$$

$$\Delta t_{tot} = L\left(\frac{1}{c-v}+\frac{1}{c+v}\right)$$

$$\boxed{\Delta t_{tot}} = 2\frac{Lc}{c^2-v^2} = \boxed{2\frac{L/c}{1-v^2/c^2}}$$

mas, do cálculo anterior:

$$\Delta t_{tot} = L\left(\frac{1}{c-v}+\frac{1}{c+v}\right)$$

$$\boxed{\Delta t_{tot}} = 2\frac{Lc}{c^2-v^2} = \boxed{2\frac{L/c}{1-v^2/c^2}}$$

mas, do cálculo anterior (*):

$$\Delta t_{tot} = \frac{1}{\sqrt{1-v^2/c^2}}\Delta t_o \qquad \Delta t_o = \frac{2L_o}{c}$$

$$\boxed{\Delta t_{tot} = \frac{2L_o/c}{\sqrt{1-v^2/c^2}}}$$

$$\Delta t_{tot} = L\left(\frac{1}{c-v}+\frac{1}{c+v}\right)$$

$$\boxed{\Delta t_{tot}} = 2\frac{Lc}{c^2-v^2} = \boxed{2\frac{L/c}{1-v^2/c^2}}$$

mas, do cálculo anterior (*):

$$\Delta t_{tot} = \frac{1}{\sqrt{1-v^2/c^2}}\Delta t_o \qquad \Delta t_o = \frac{2L_o}{c}$$

$$\boxed{\Delta t_{tot} = \frac{2L_o/c}{\sqrt{1-v^2/c^2}}} \quad =$$

então

$$2\frac{L_o/c}{\sqrt{1-v^2/c^2}} = 2\frac{L/c}{1-v^2/c^2}$$

$$L = L_o\sqrt{1-v^2/c^2}$$

$$\gamma \equiv \frac{1}{\sqrt{1-v^2/c^2}}$$

$$L = \frac{1}{\gamma}L_o$$

$$\gamma > 1$$

$$L < L_o$$

$$\Delta t_1 > \Delta t_o$$

$$L < L_o$$

Dilatação Temporal

$$\Delta t_1 > \Delta t_o$$

$$L < L_o$$

Contração Espacial

TRANSFORMAÇÕES DE LORENTZ

$$t' = \gamma \left(t - \frac{v \cdot x}{c^2} \right)$$

$$x' = \gamma \left(x - v \cdot t \right)$$

$$y' = y$$

$$z' = z$$

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}}$$

TRANSFORMAÇÕES DE LORENTZ

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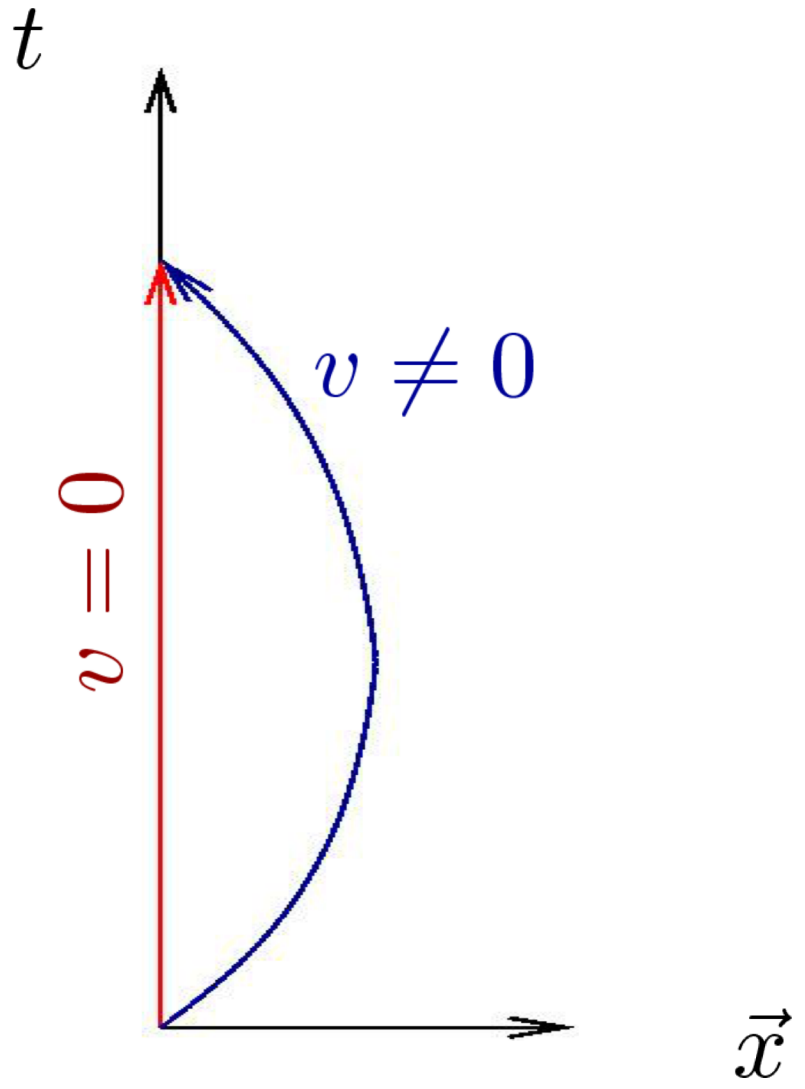
Efeitos mensuráveis?

- Aceleradores de partículas
- Raios cósmicos
- Sistema de posicionamento global (GPS)

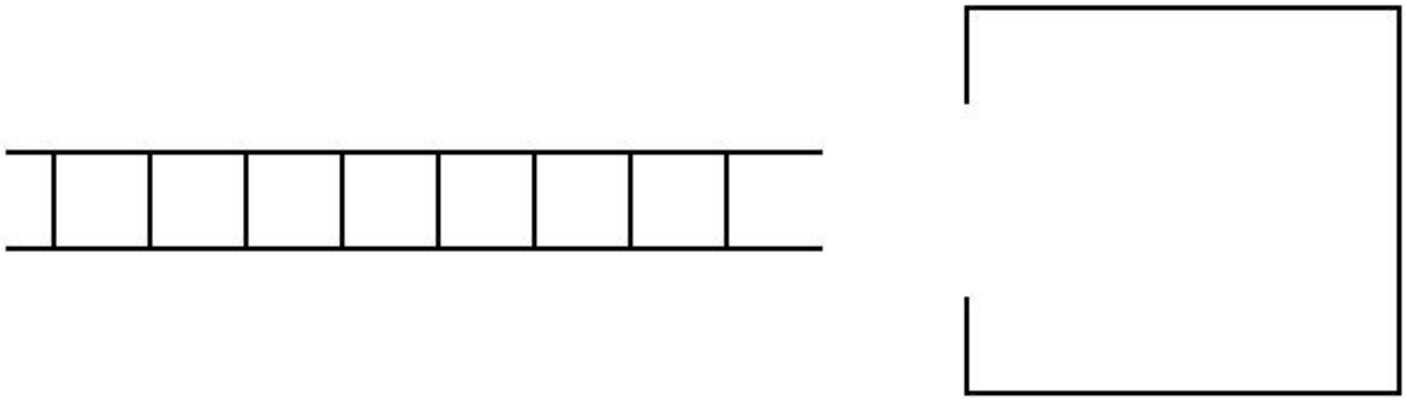
Alguns paradoxos interessantes...

Gêmeos

$$\Delta t_1 = \gamma \Delta t_o$$



Escada e o celeiro



Relatividade geral

O elevador de Einstein e o Princípio da equivalência

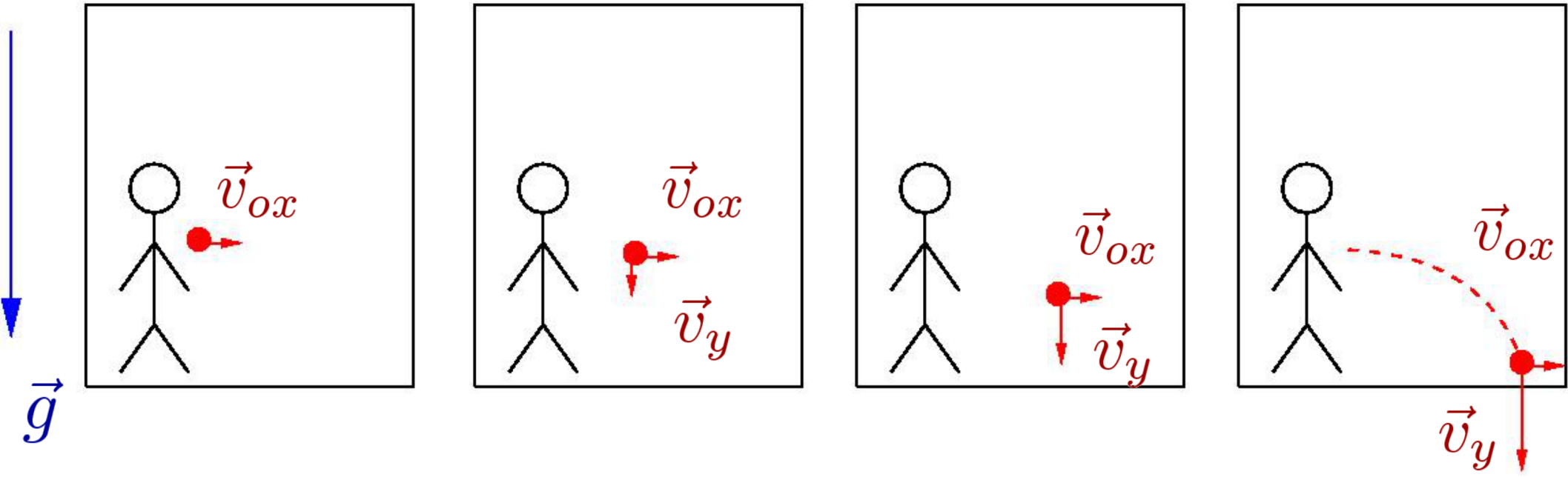
$$m_i = m_g$$

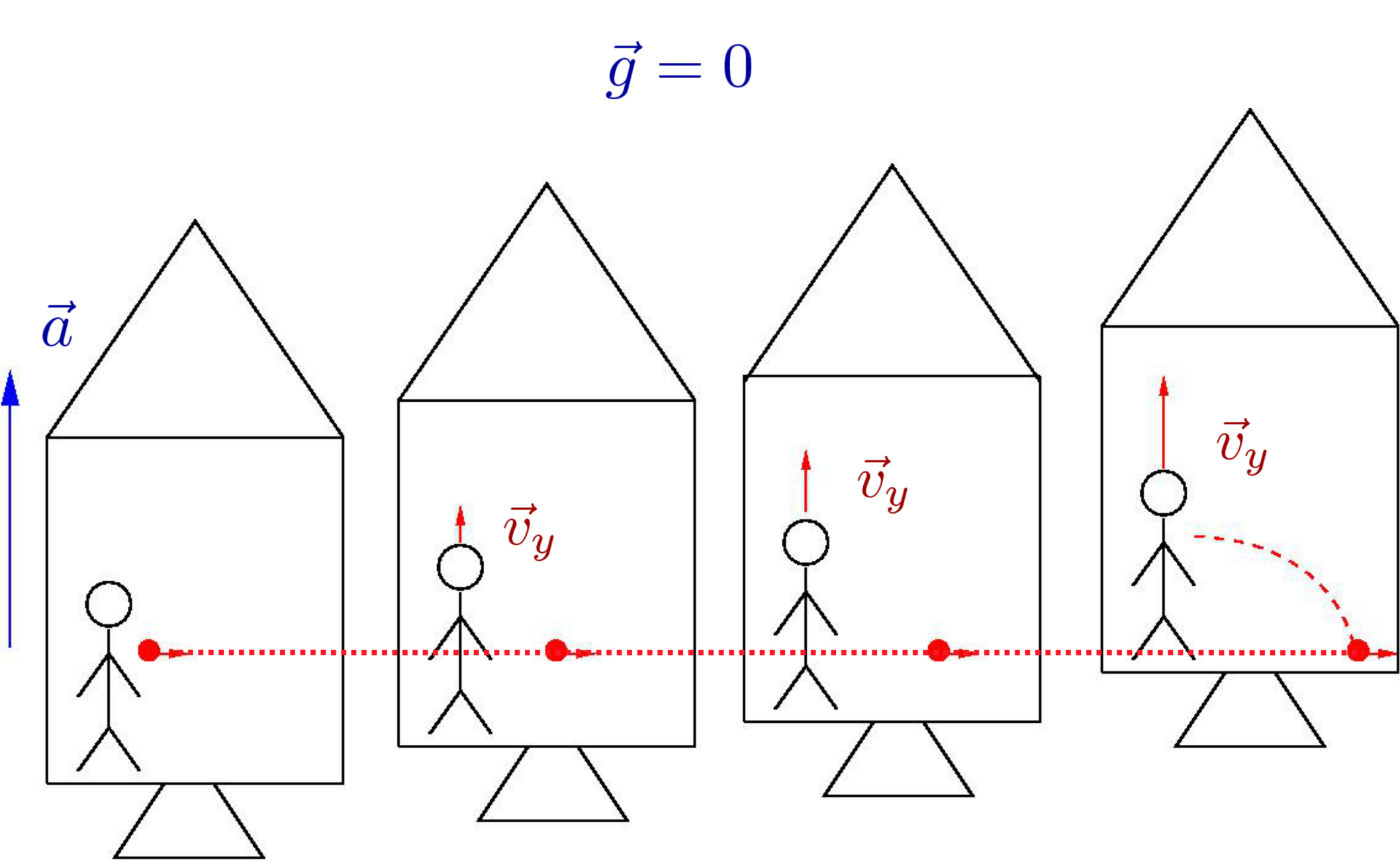
$$F = m_i \cdot a$$

$$F_g = G\frac{m_{g1} \cdot m_{g2}}{r^2}$$

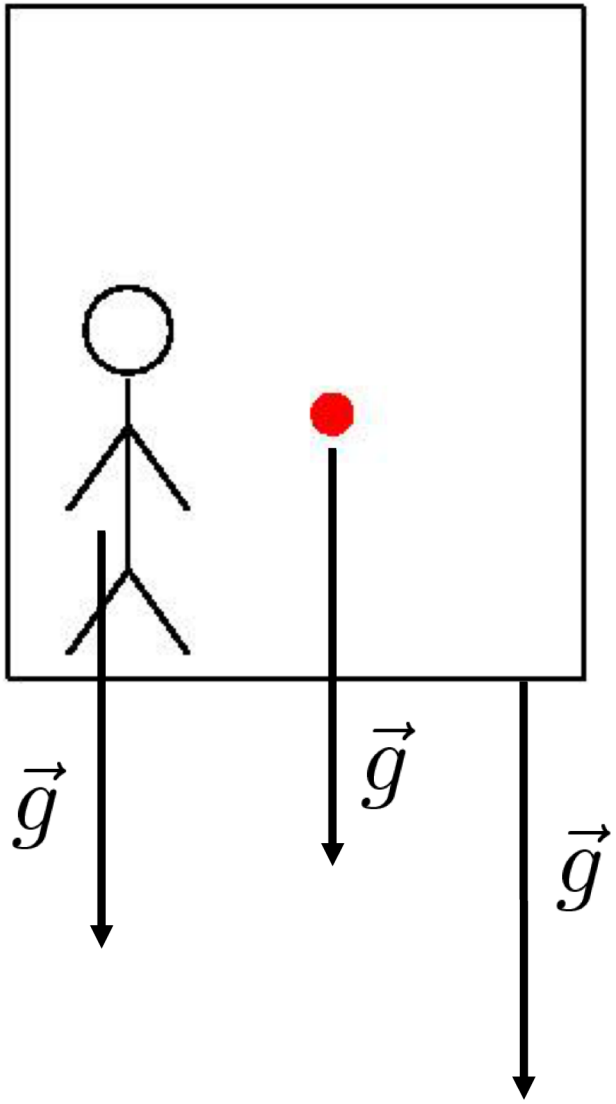
$$\left(F_E = k\frac{q_1 \cdot q_2}{r^2}\right)$$

$$\vec{v} = 0 \qquad \vec{a} = 0$$

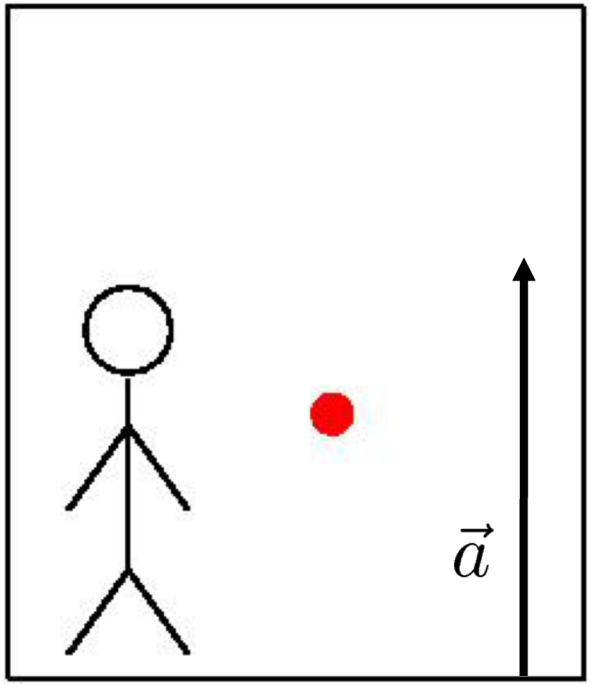



$\vec{g} = 0$
$\vec{a}$
$\vec{v}_y$
$\vec{v}_y$
$\vec{v}_y$

$\vec{g} \neq 0 \qquad \vec{a} = 0$



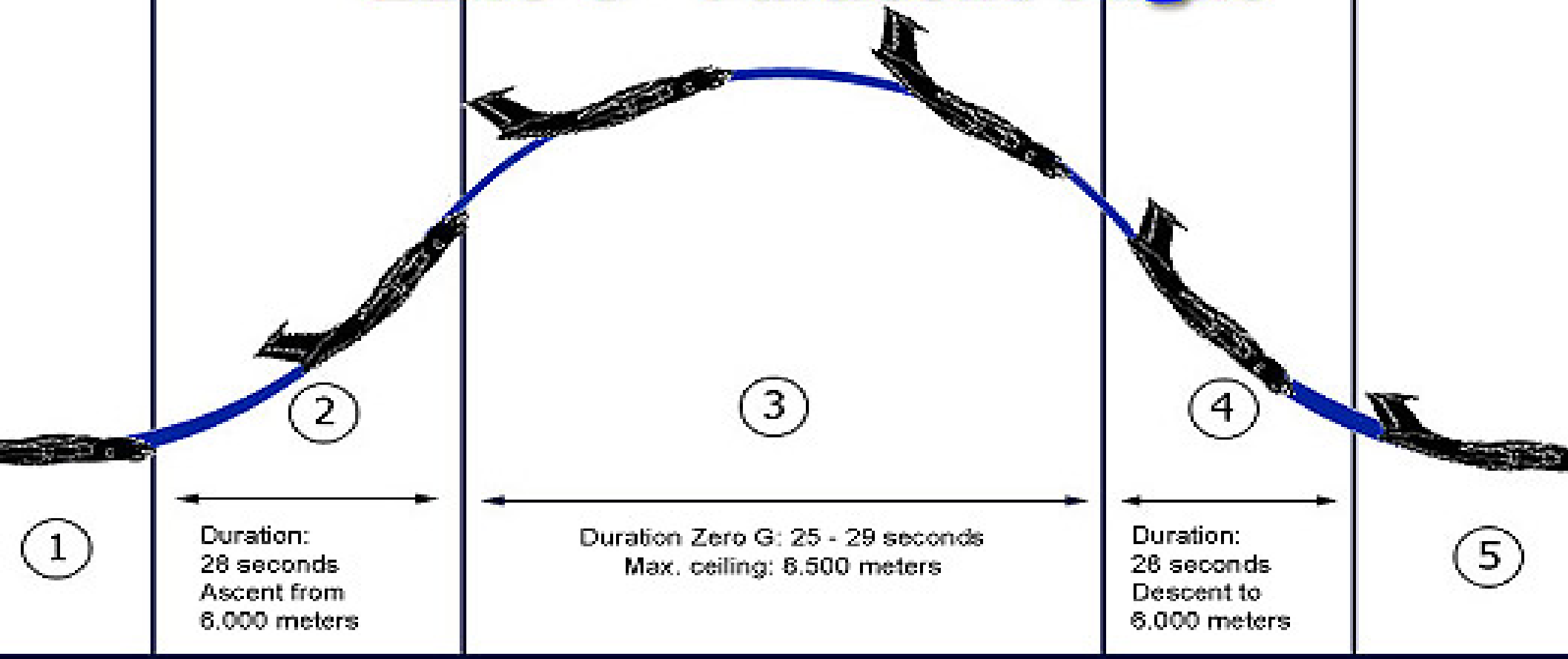
$$\vec{g} = 0 \qquad \vec{a} \neq 0$$



$$m_i = m_g$$

FORÇA DE INÉRCIA ?!

ZERO G - Parabolic Flight



HOW DOES IT WORK?

1. Horizontal Flight: 1G

2. Pull up Phase: 2G

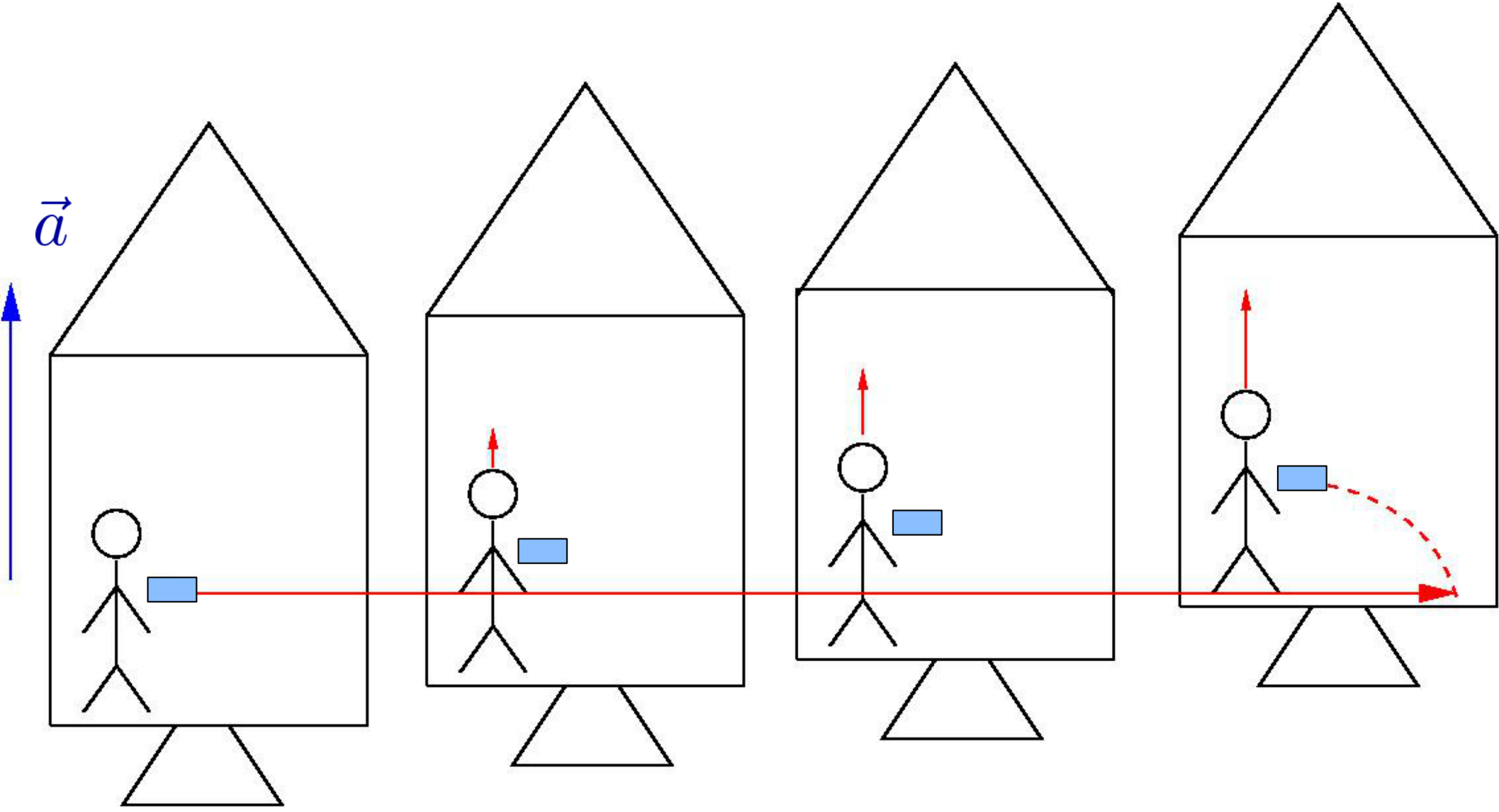
3. Zero G Injection Phase: 0G

4. Pull out Phase: 2G

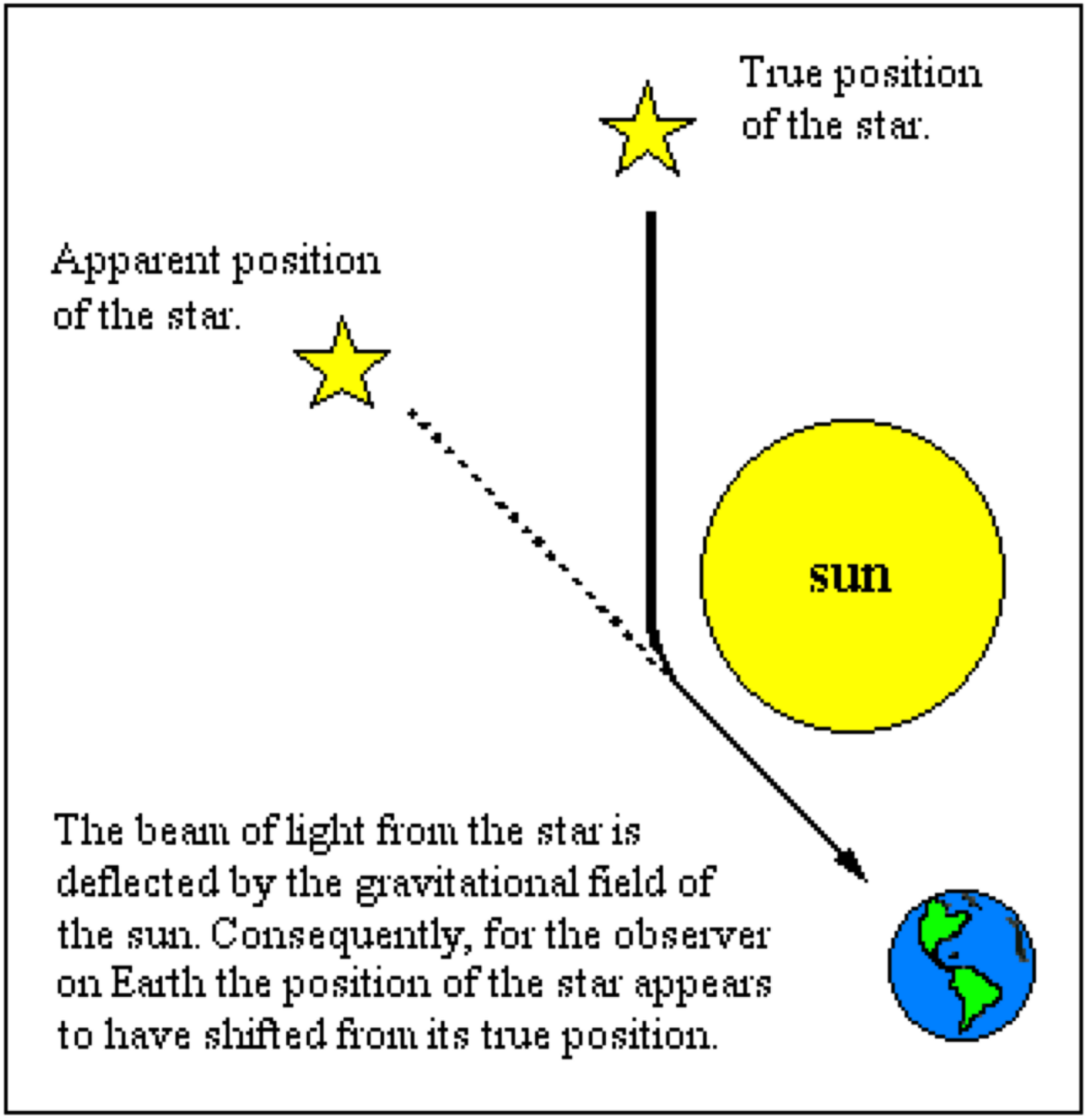
5. Normal Flight: 1G

26 / abril / 2007

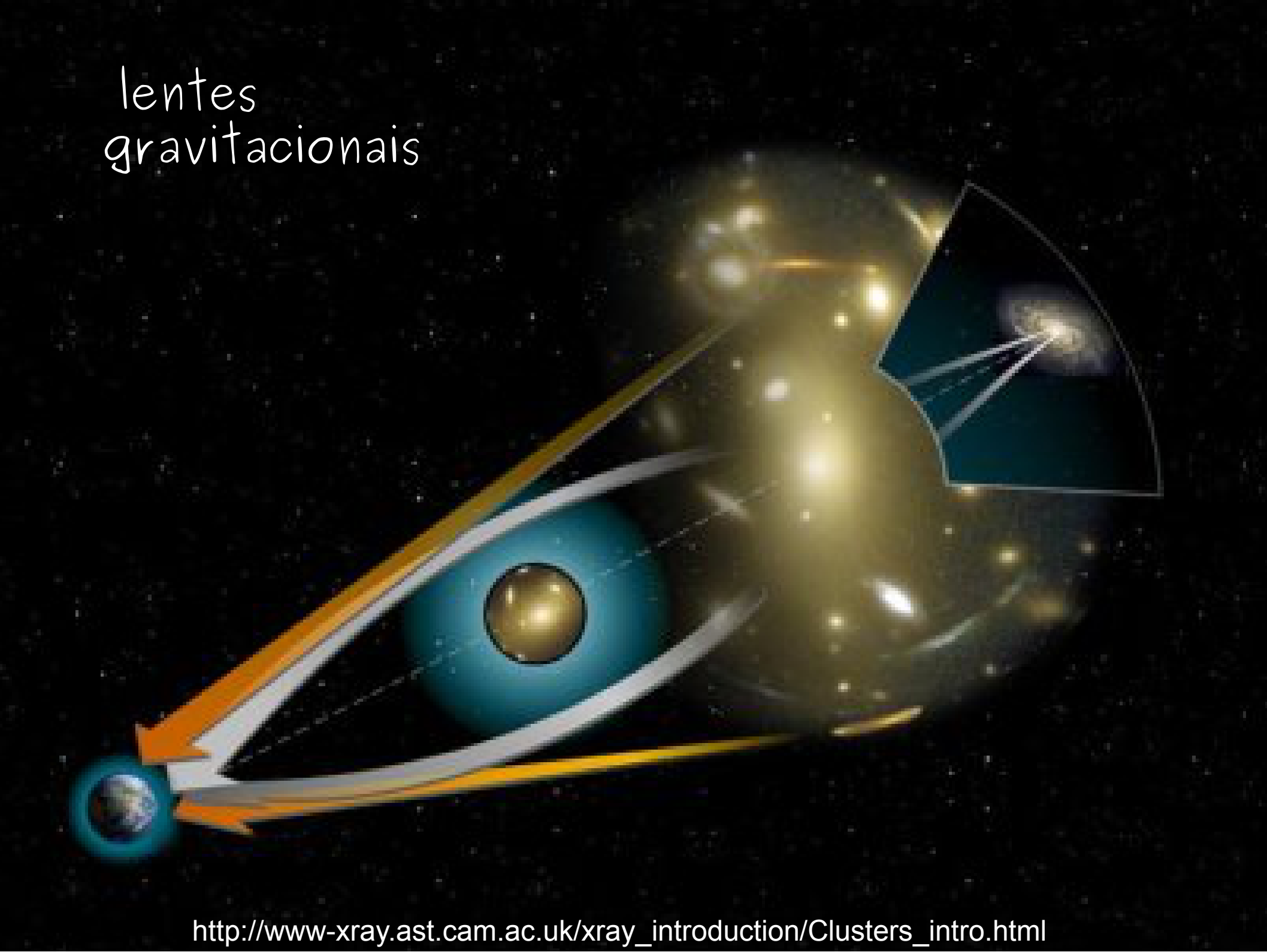
Se as descri**ções** s**ão**
mesmo completamente
equivalentes,
ent**ão**...


$\vec{a}$

- Desvio da luz pelo sol



lentes gravitacionais



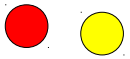
http://www-xray.ast.cam.ac.uk/xray_introduction/Clusters_intro.html

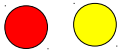
Galaxy Cluster Abell 2218 **HST** • WFPC2

NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

http://ircamera.as.arizona.edu/NatSci102/NatSci102/lectures/galaxydist.htm

Vamos ver como esta ideia pode ser **ú**til em casos mais concretos...

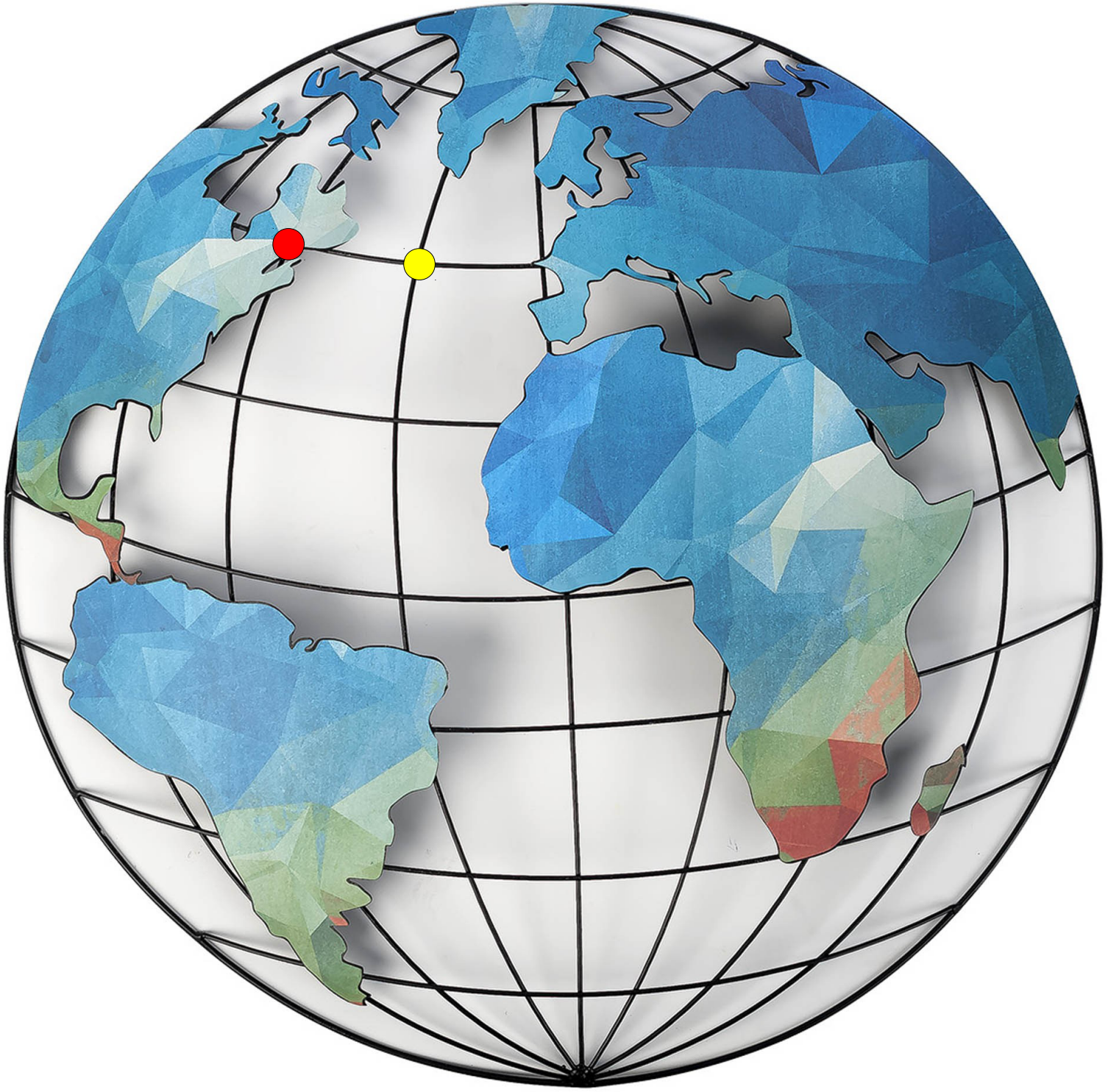


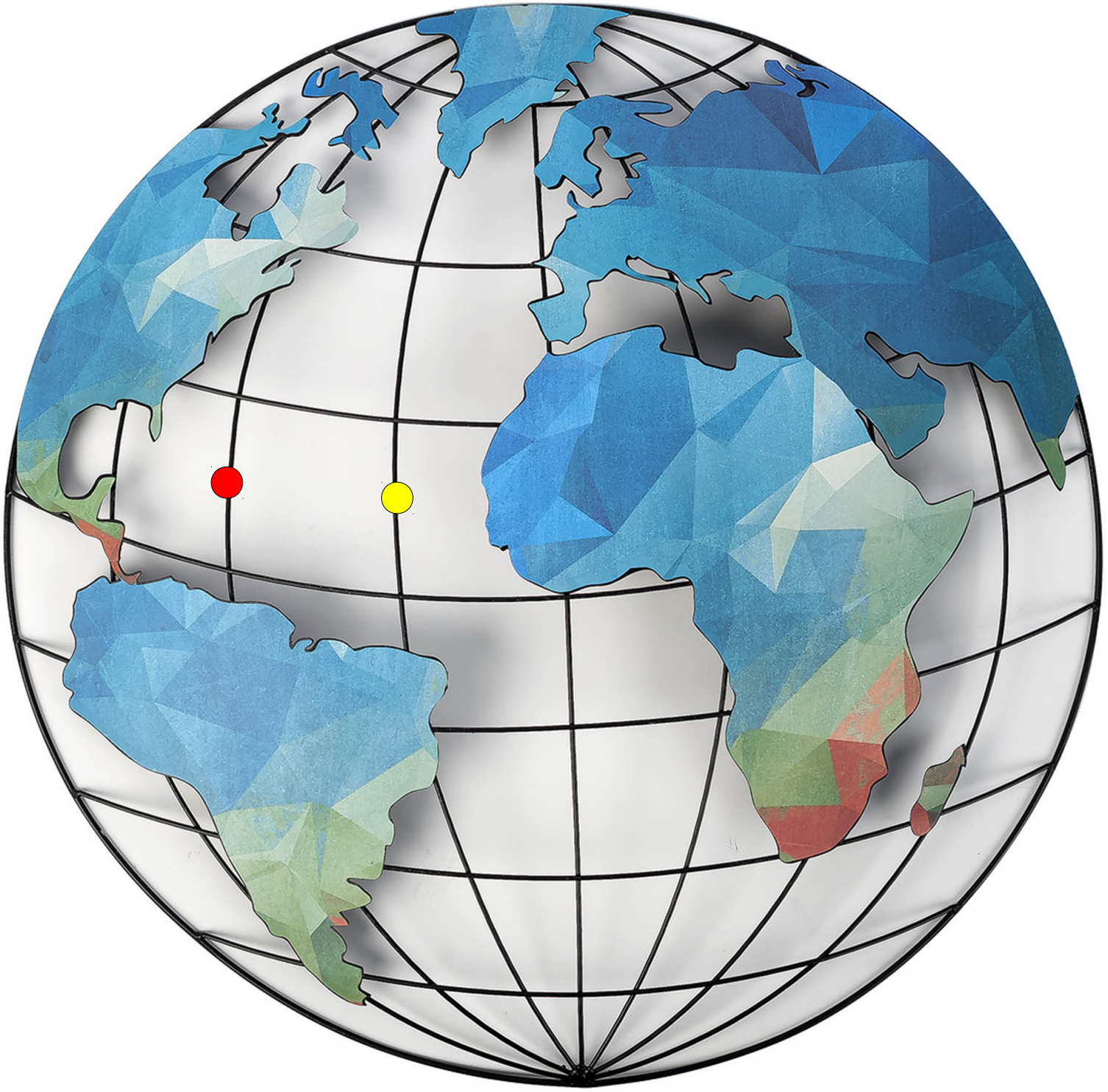


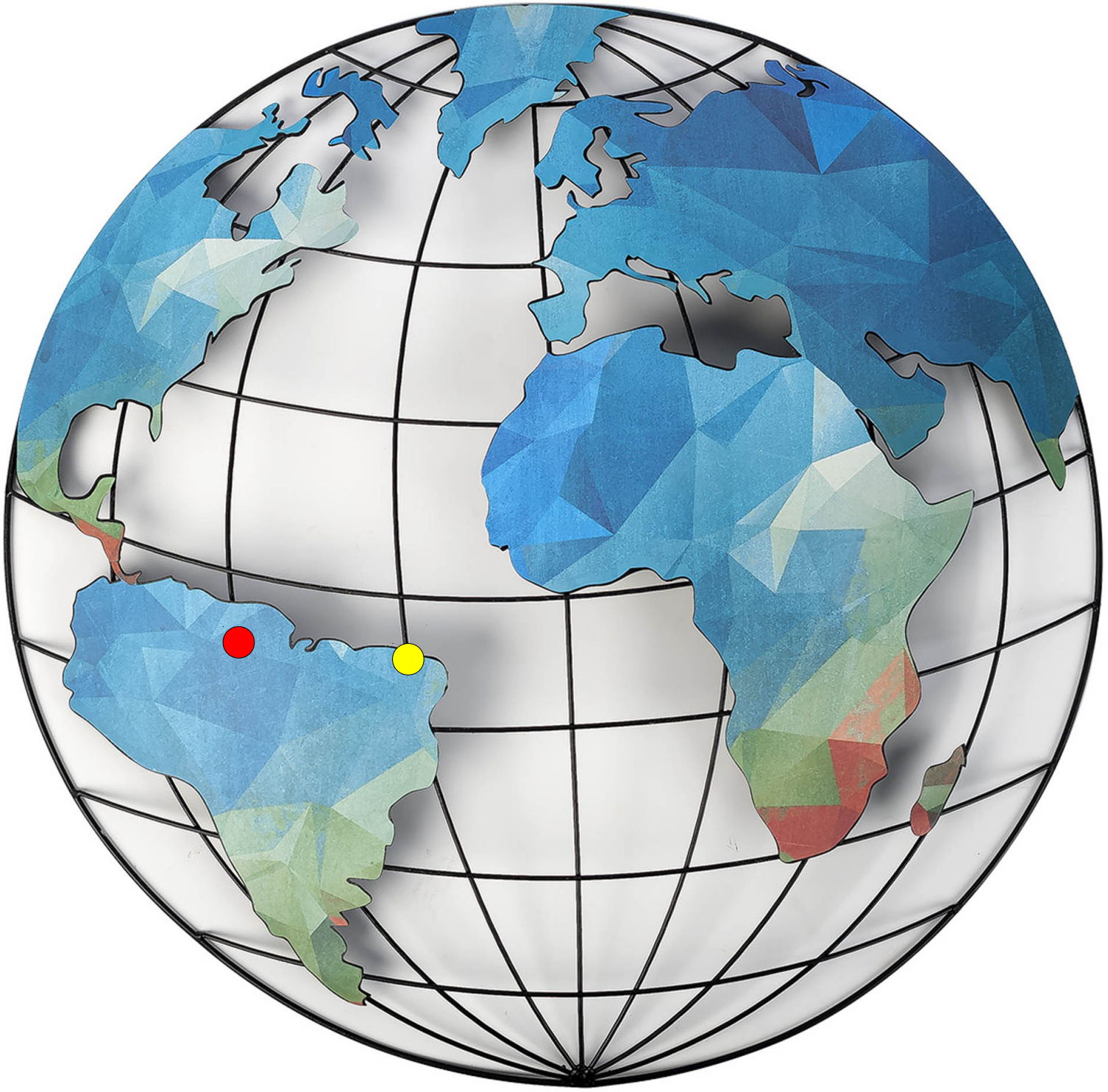
As partículas se afastam e depois de aproximam.

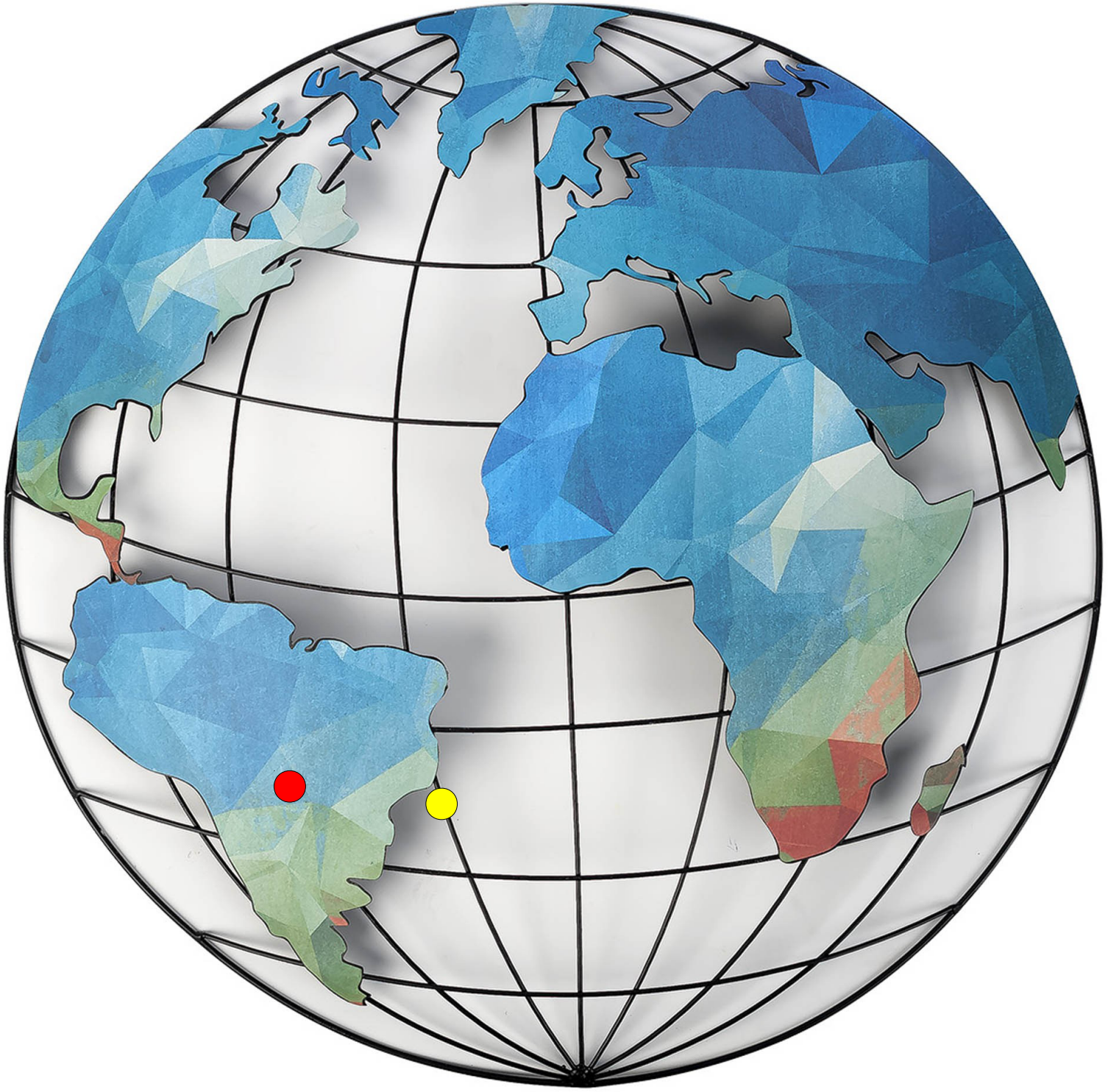
Deve, portanto, haver uma força de interação entre elas!

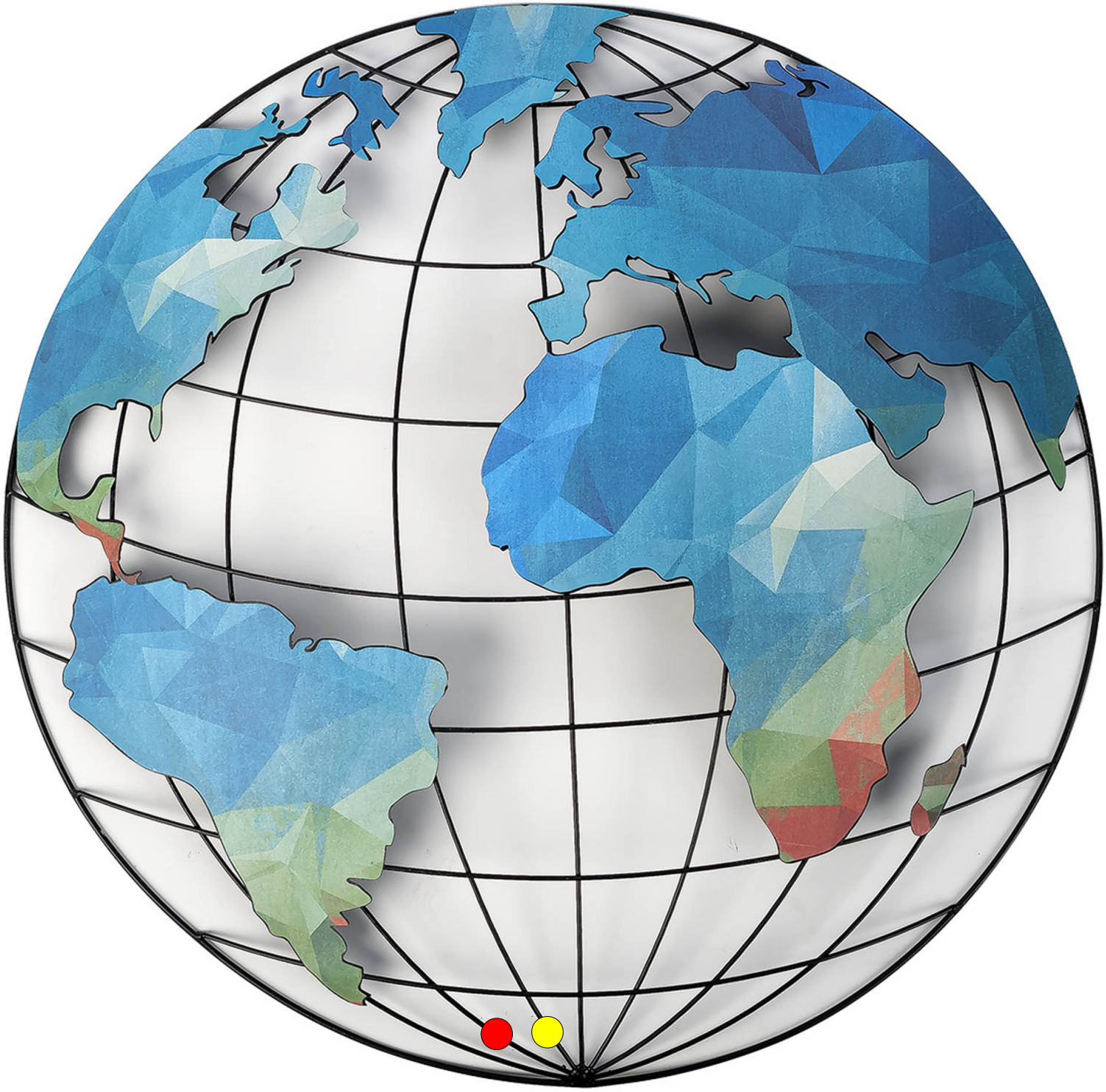
Mas existe uma maneira mais
fácil de explicar estas
deformações nas trajetórias,
SEM falar de força gravitacional
nem aceleração!



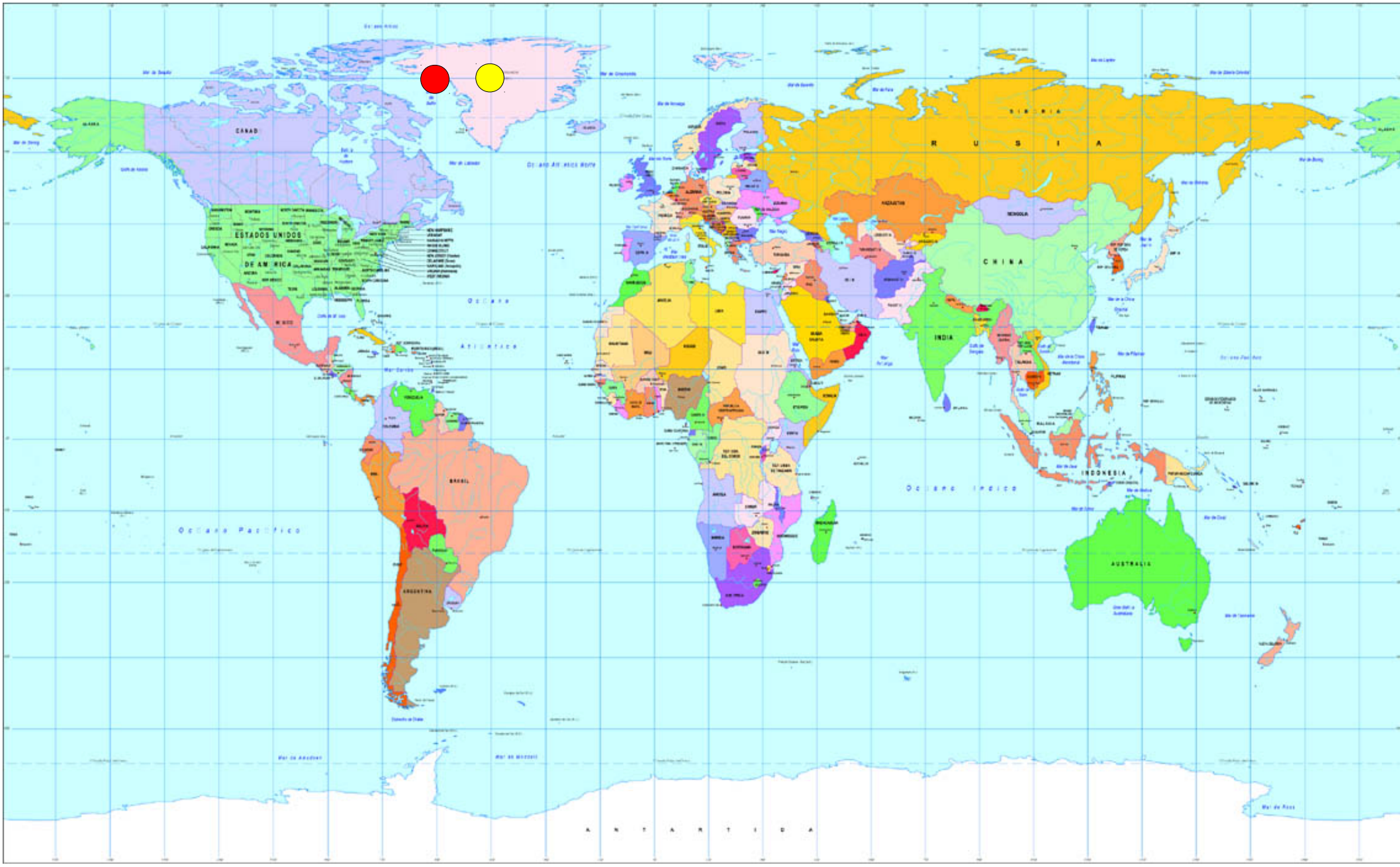








Uma outra opção
é mudar a geometria,
ou seja, nossa medida
de distância !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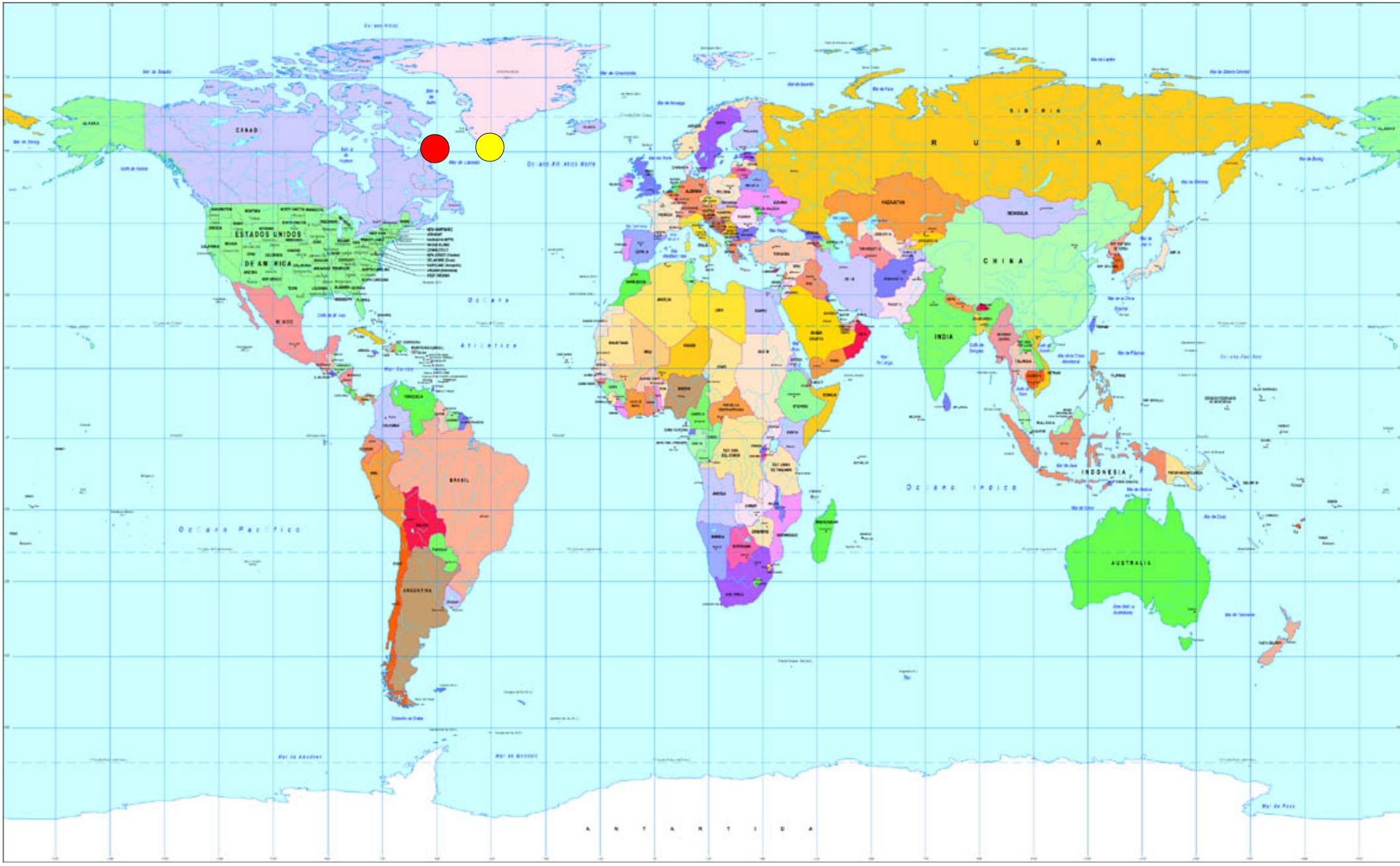
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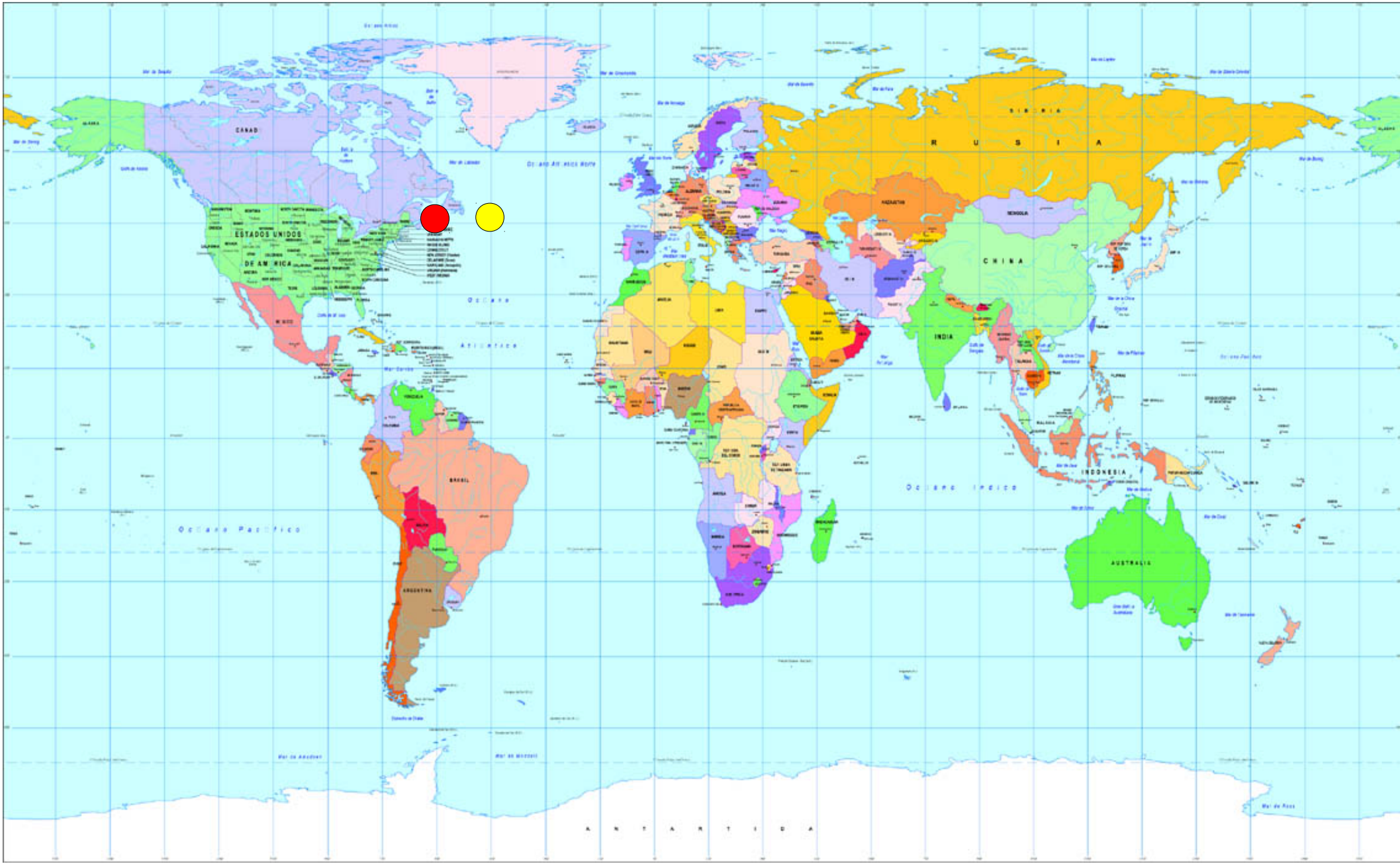
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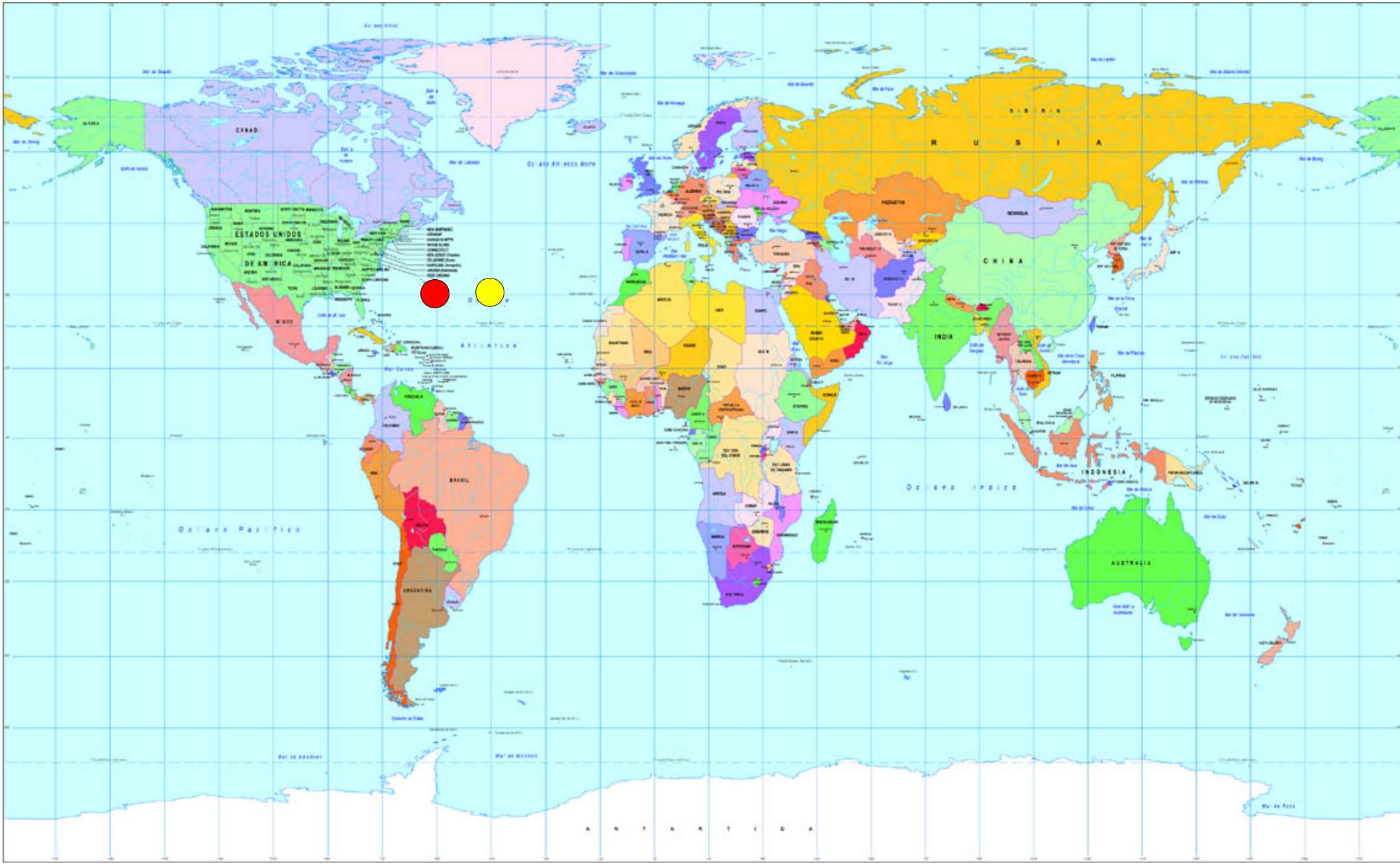
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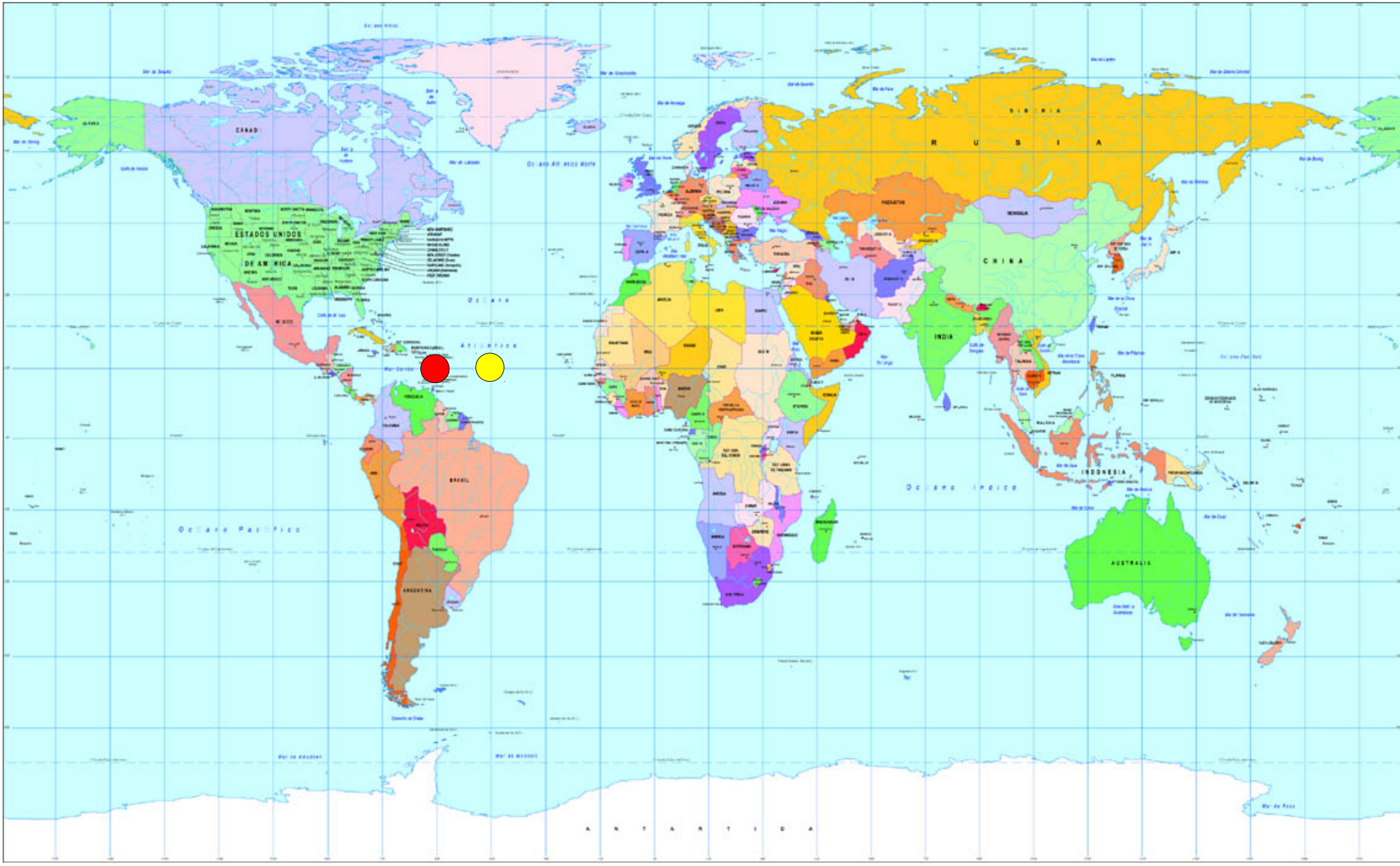
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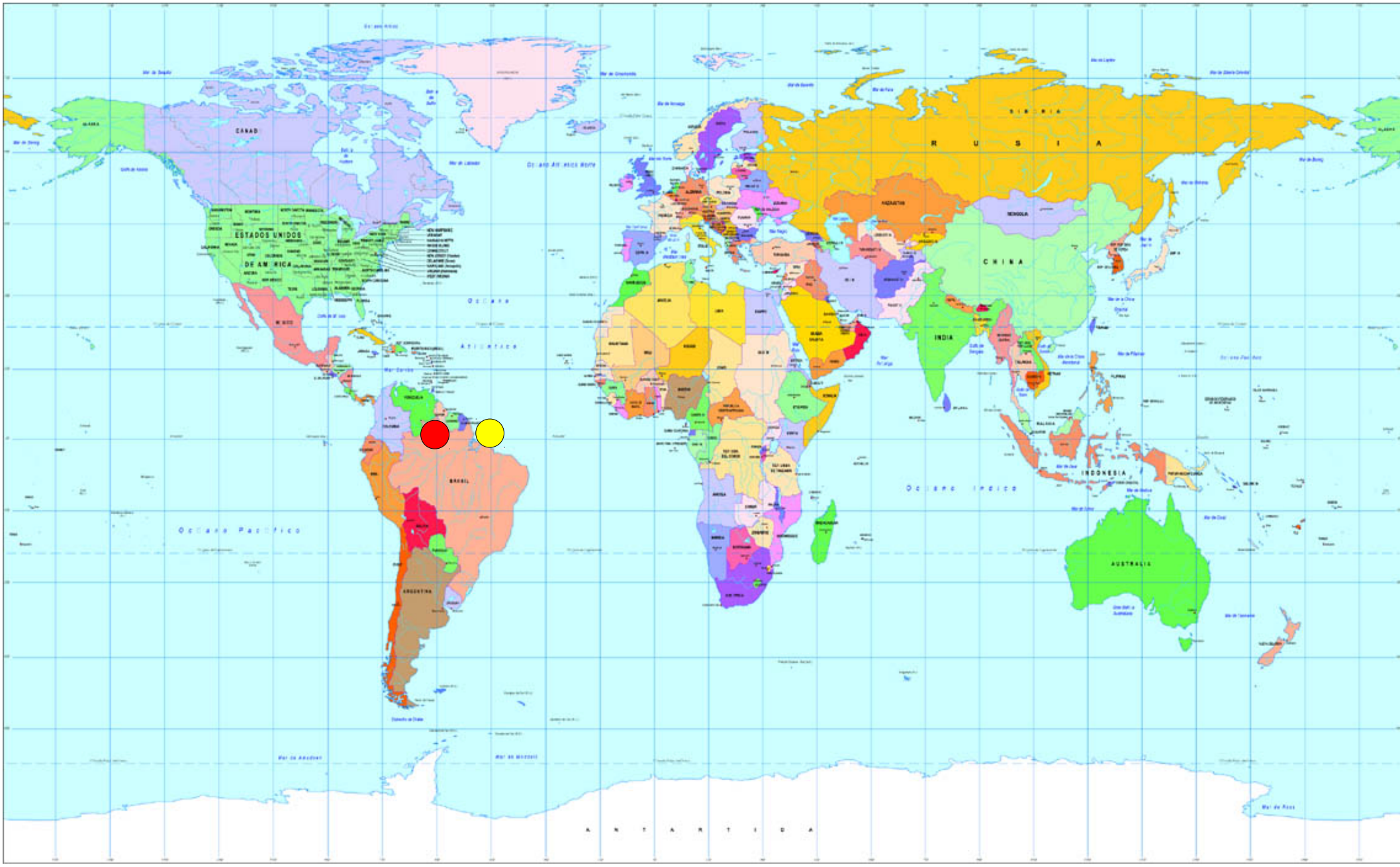
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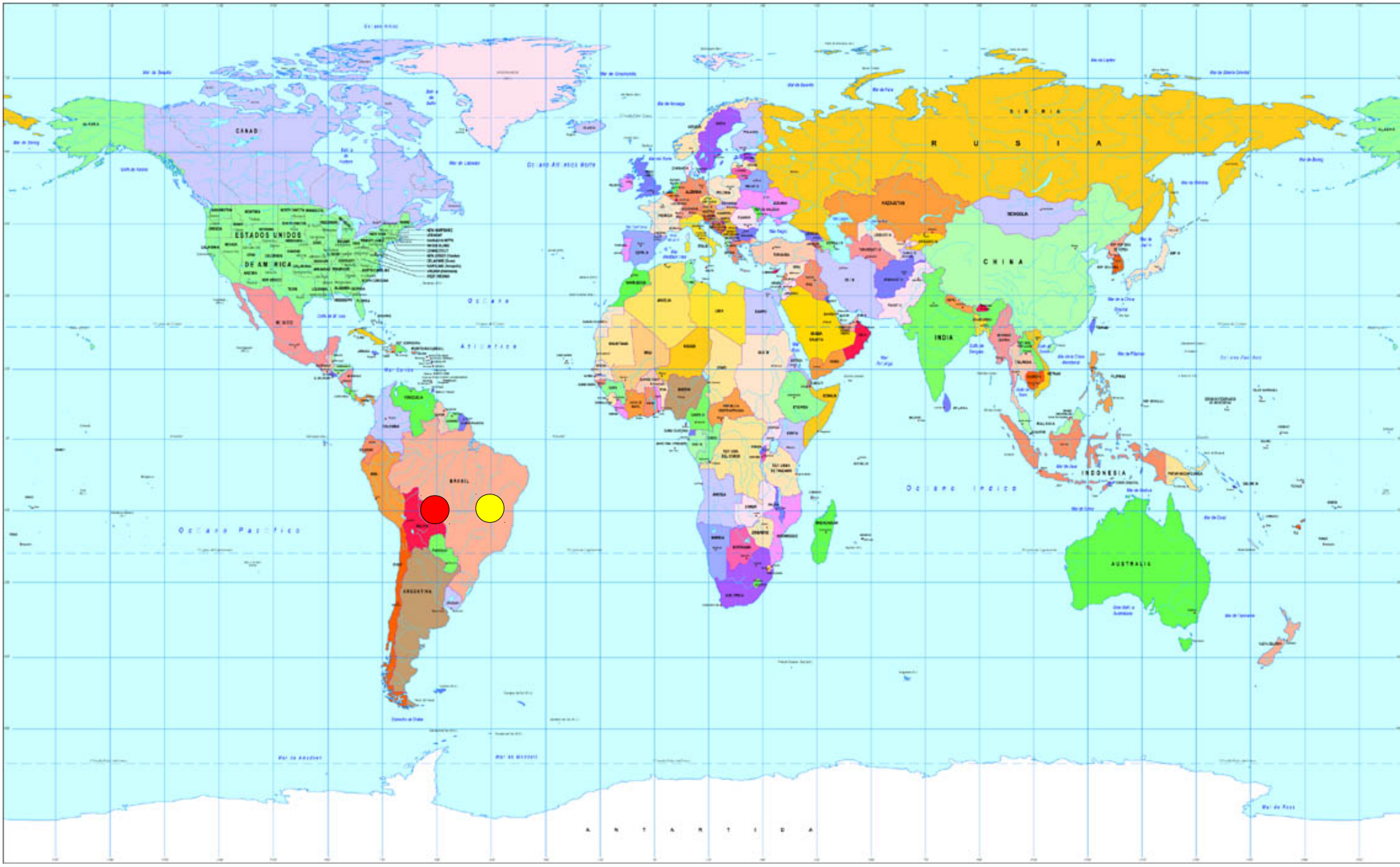
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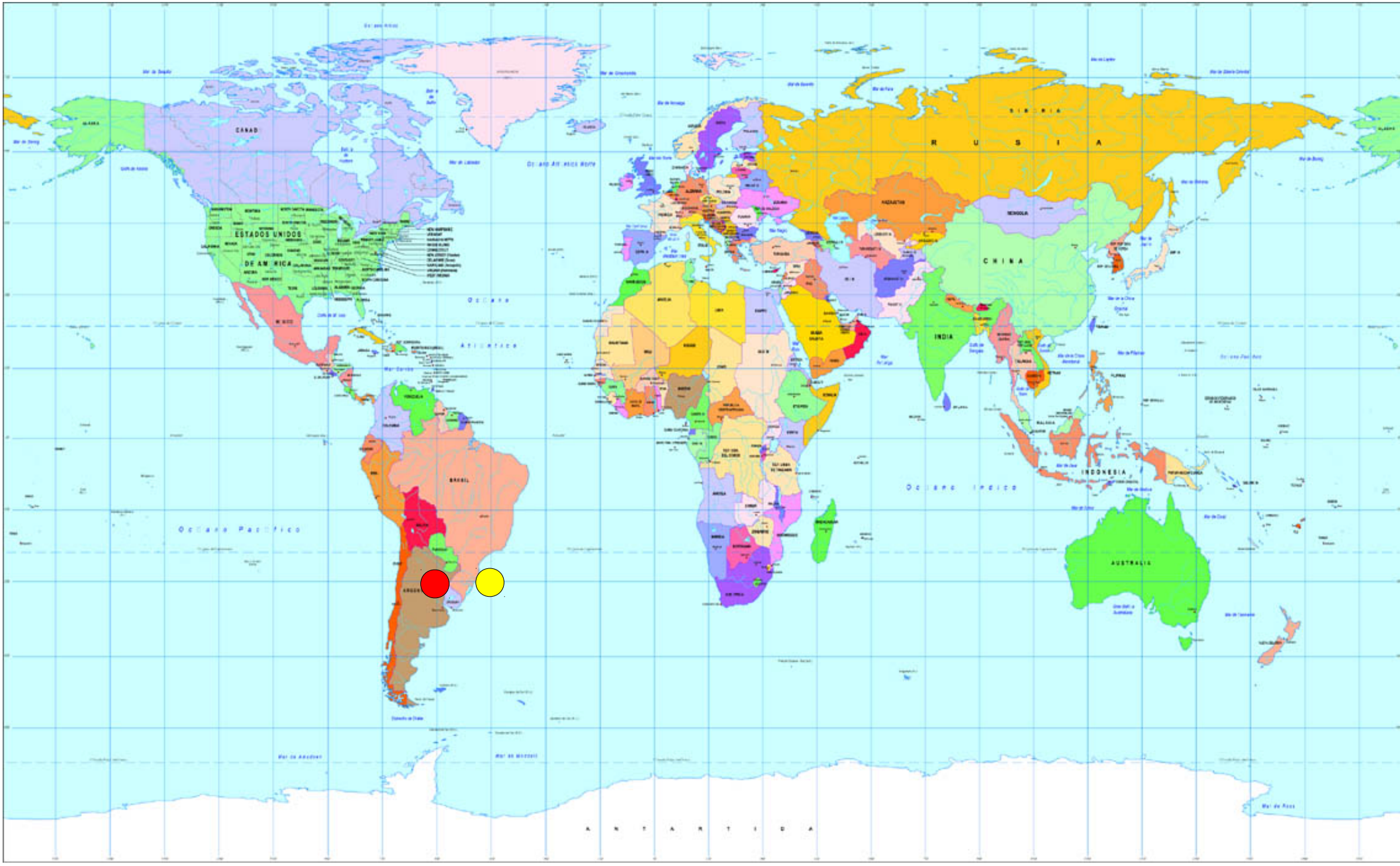
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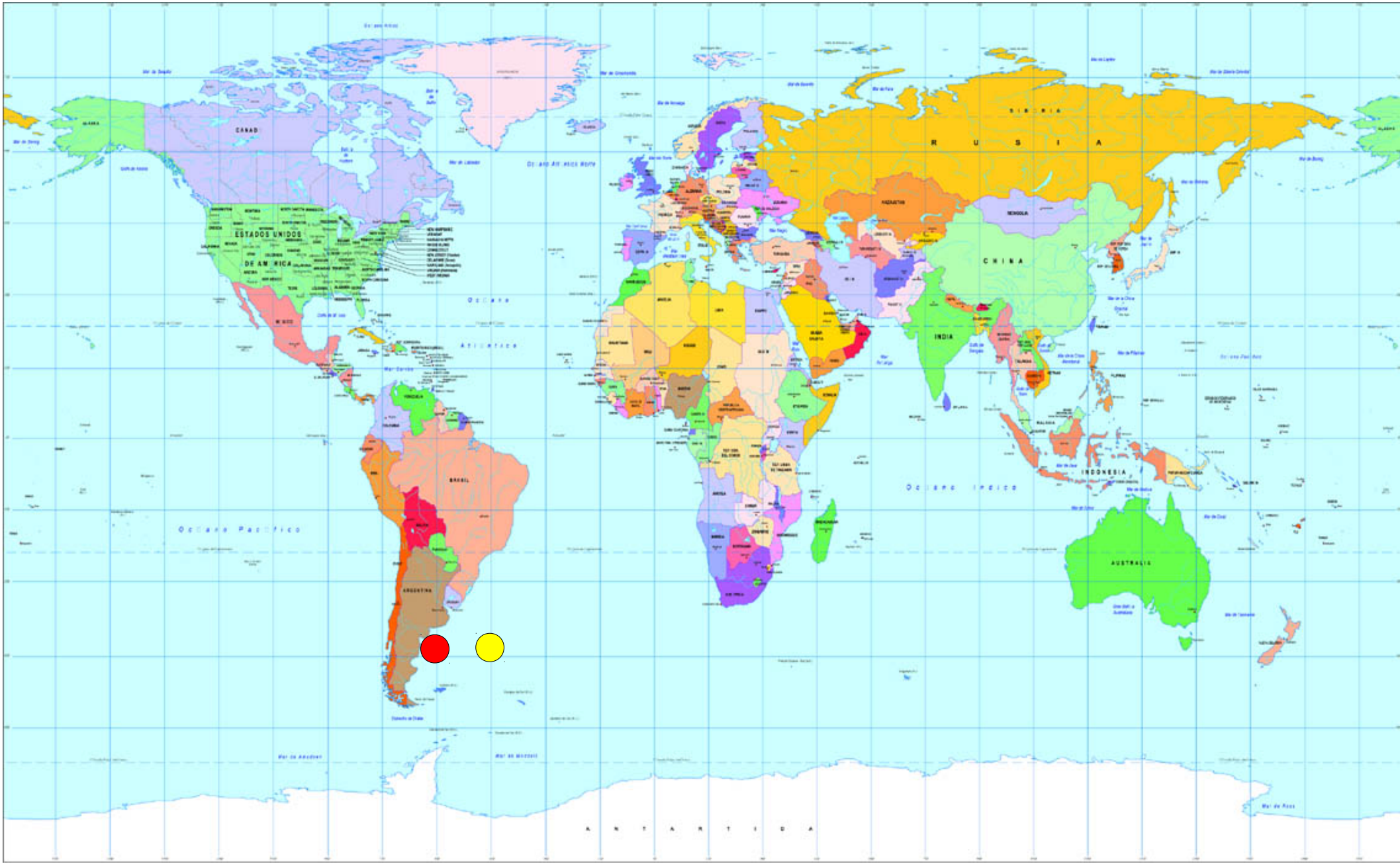
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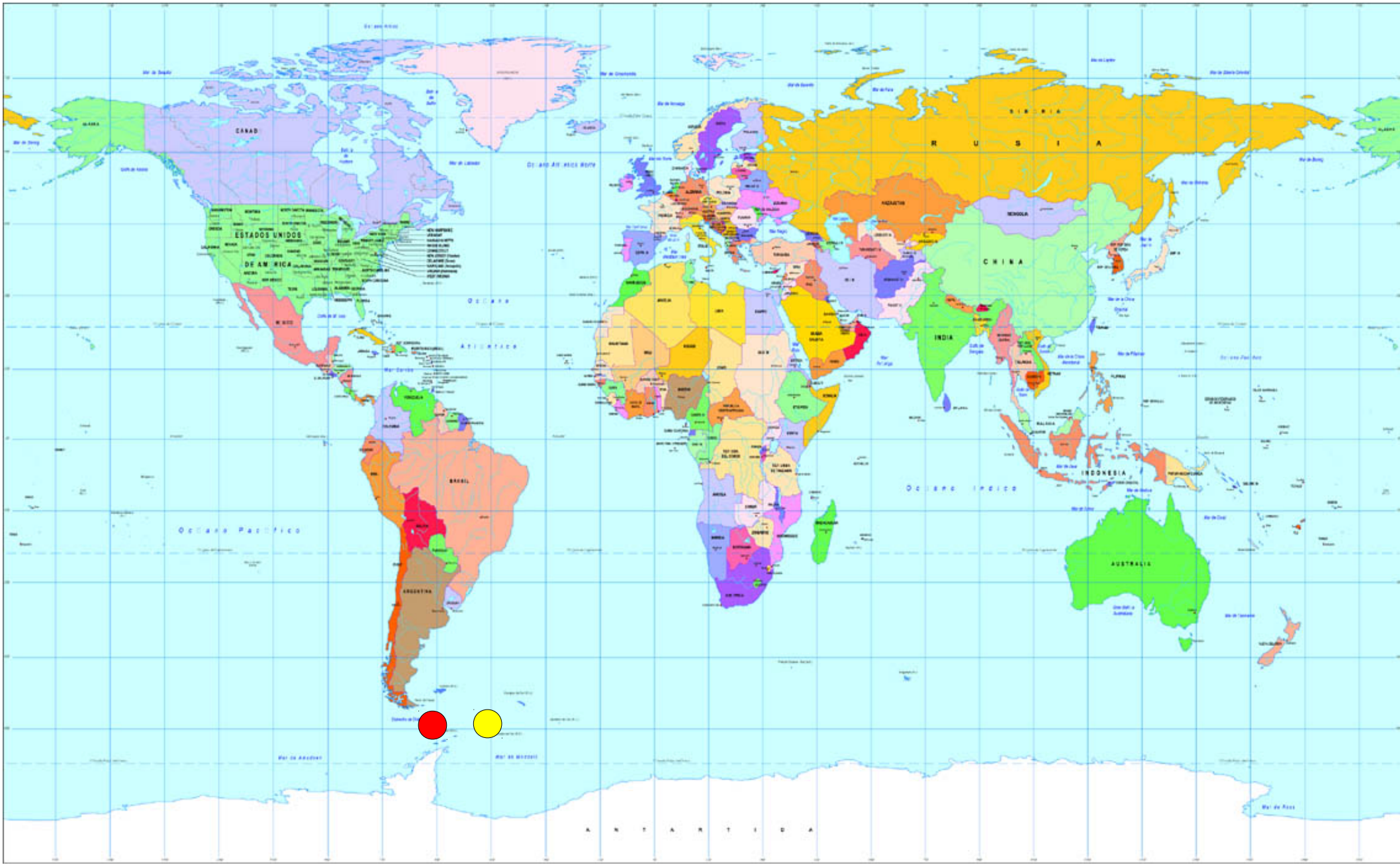
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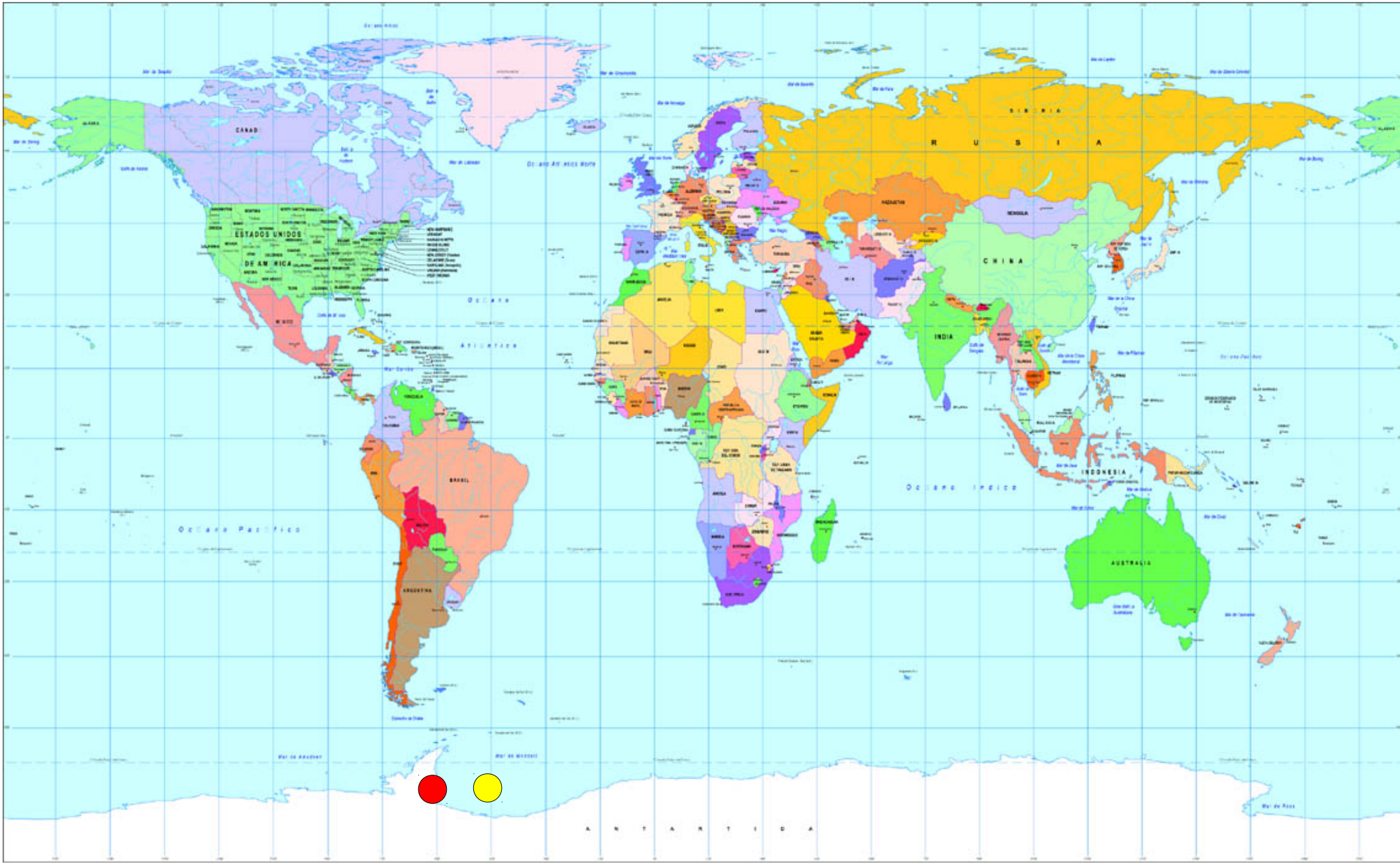
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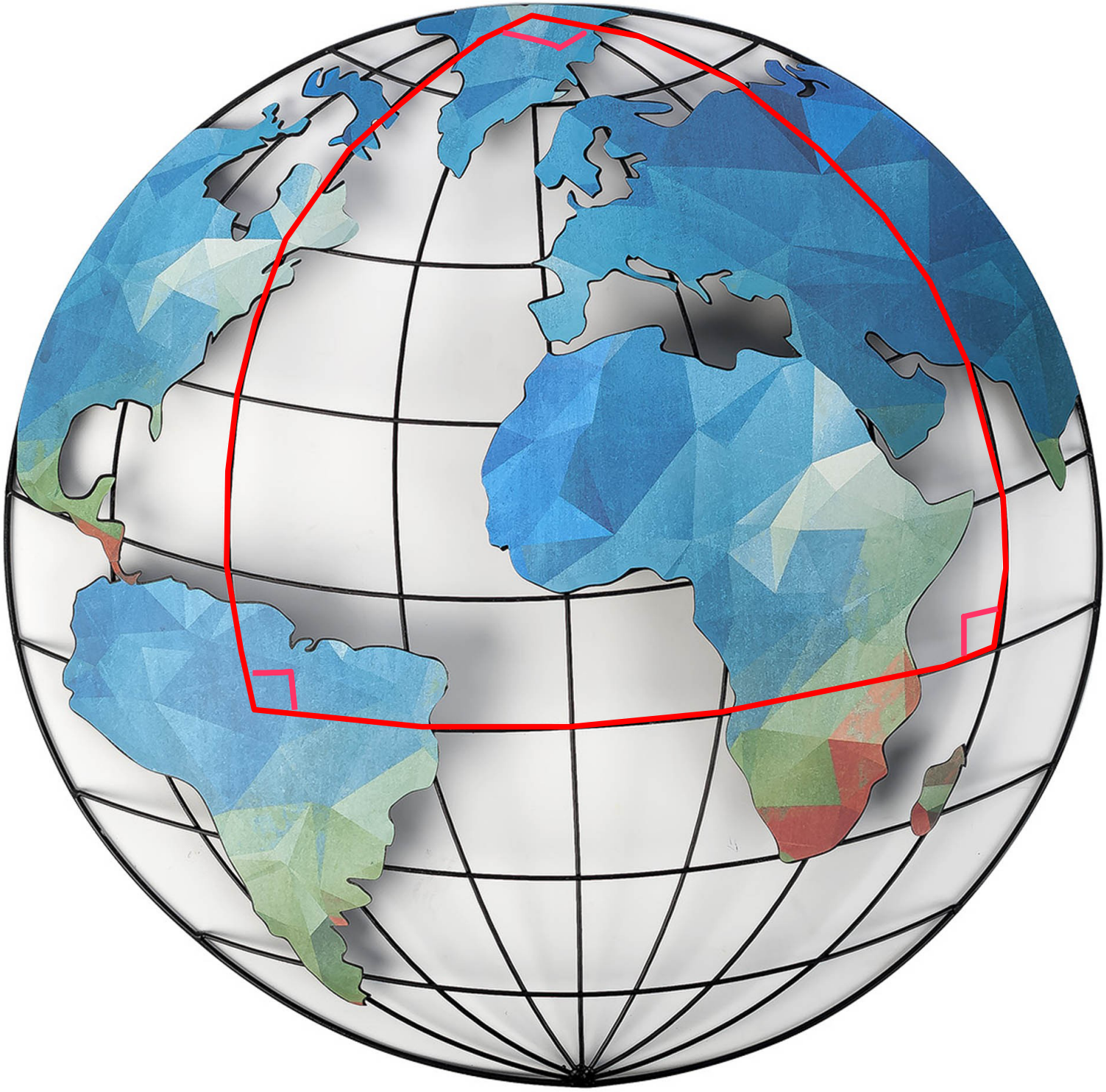
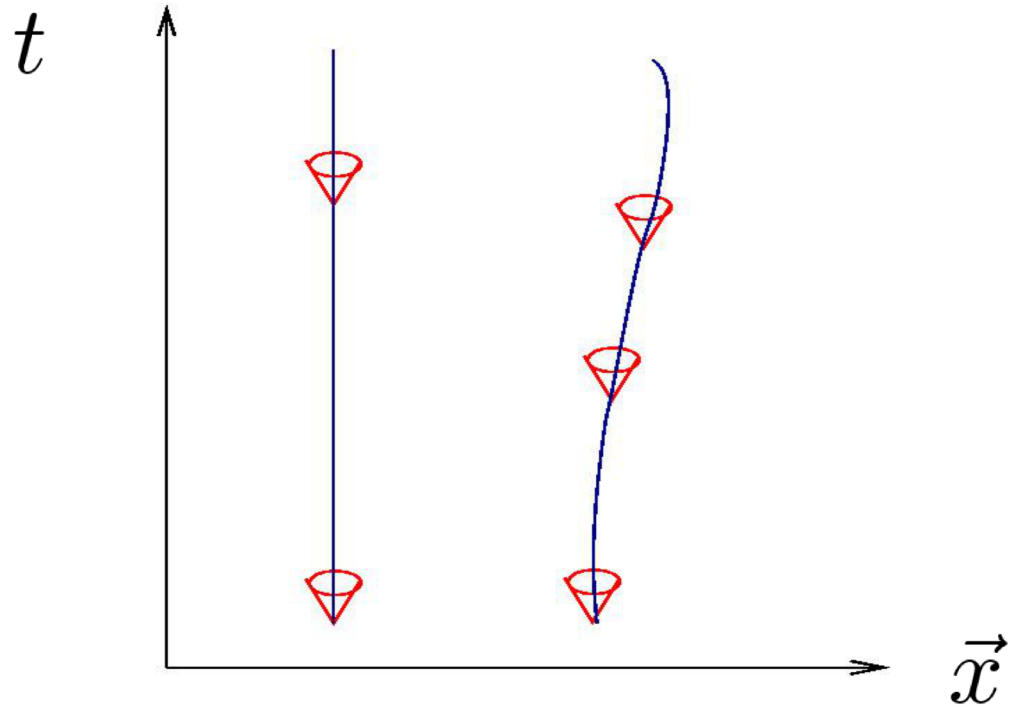
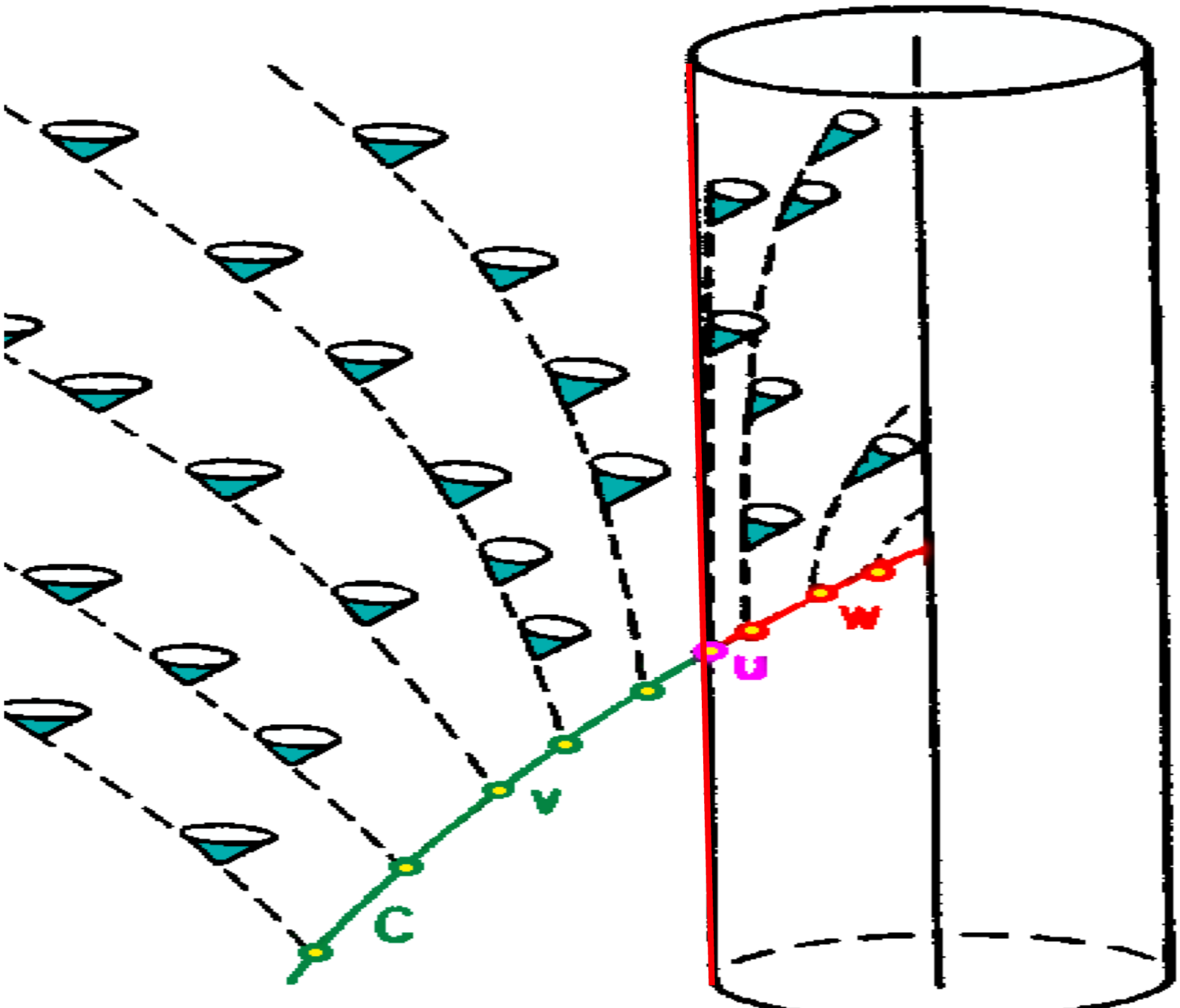
Nesta nova geometria,
não há
necessidade de introduzir
uma força gravitacional!



Estes efeitos são maiores
próximos a grandes massas

- Buracos negros
  - Clássicos
  - Relativísticos
  - Origem estelar ou cosmológica

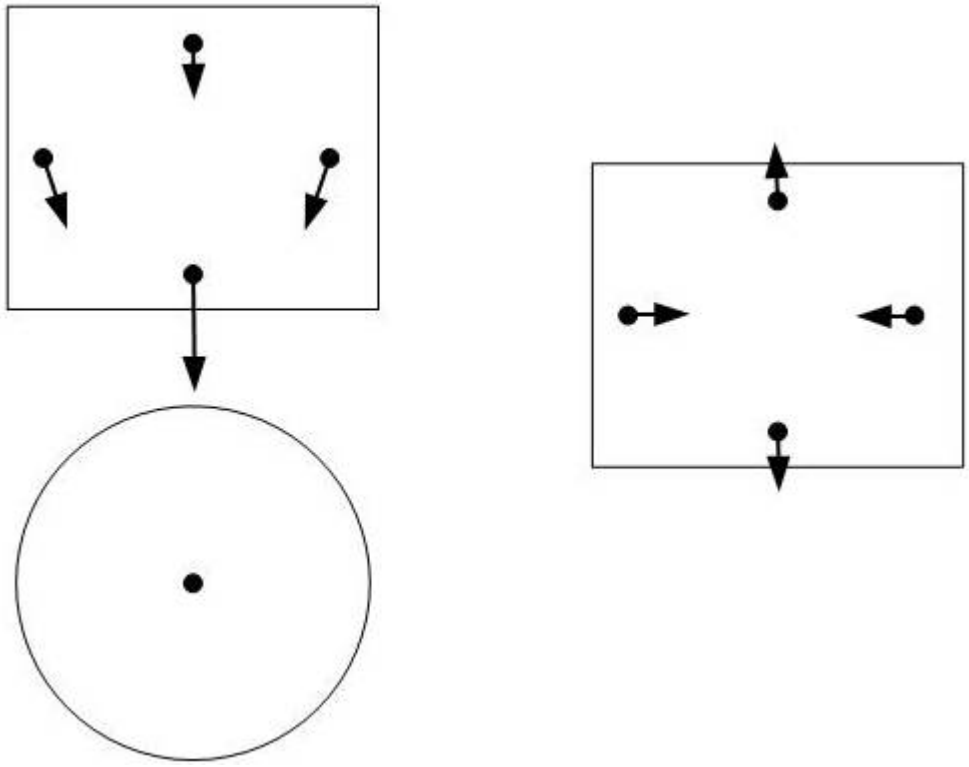




http://www.phy.syr.edu/courses/modules/LIGHTCONE/schwarzschild.html

Horizonte de eventos
não é uma barreira!

- efeitos de maré

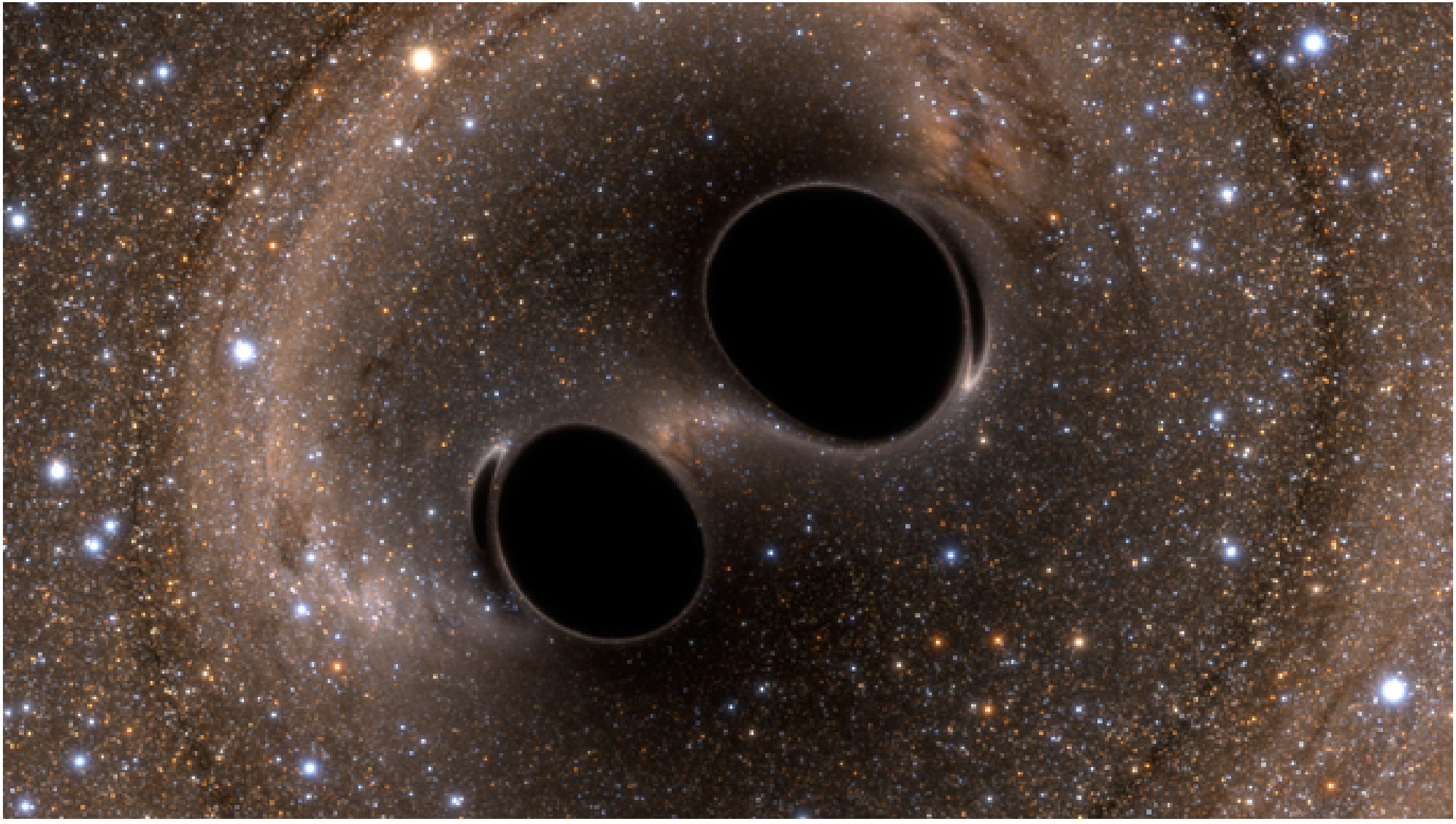


http://quantumrelativity.calsci.com/Physics/SpaceAndTime2.html

Radiação Hawking

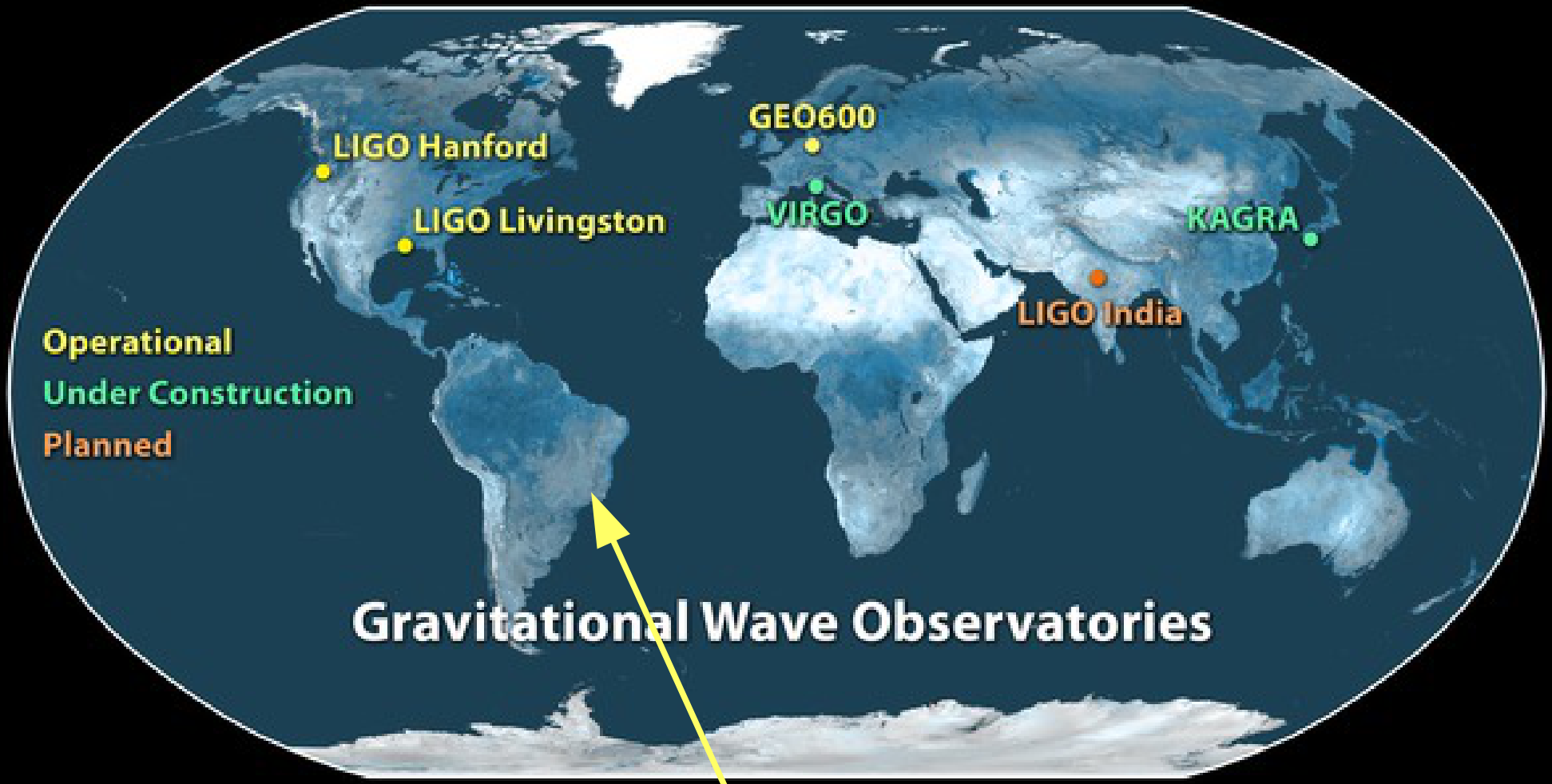
- criação de pares partícula–antipartícula
- taxa de criação $\sim \frac{1}{M_{BN}}$
- final ($M \to 0$): ?!?!?!

LASER INTERFEROMETER GRAVITATIONAL-WAVE OBSERVATORY

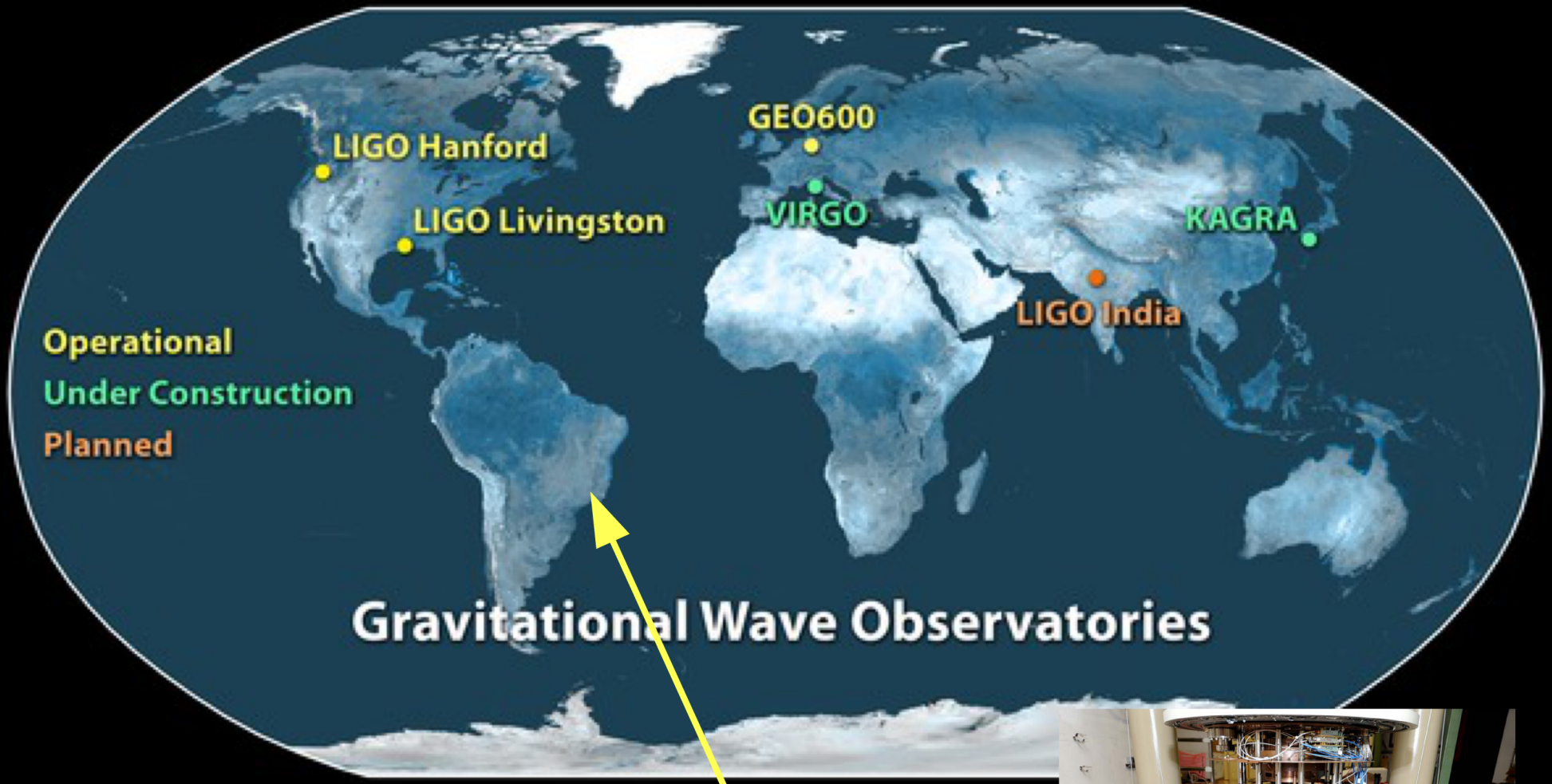


Participação Brasileira

- Odylio Denys de Aguiar (Pesquisador Titular III, servidor do INPE)
- Marcos André Okada (Tecnico III, servidor do INPE)
- César Augusto Costa (Pos-doc, bolsista do CNPq)
- Márcio Constâncio Jr (aluno de doutorado do curso de pós-graduação em Astrofisica do INPE, bolsista da FAPESP)
- Elvis Camilo Ferreira (aluno de doutorado do curso de pós-graduação em Astrofísica do INPE, bolsista da CAPES)
- Allan Douglas dos Santos Silva (aluno de iniciação científica, bolsista da FAPESP)
- Riccardo Sturani (ICTP-SAIFR, no IFT-UNESP, bolsista "Jovem Pesquisador" da FAPESP)


LIGO Hanford
LIGO Livingston
GEO600
VIRGO
KAGRA
LIGO India
Operational
Under Construction
Planned
Gravitational Wave Observatories

Schenberg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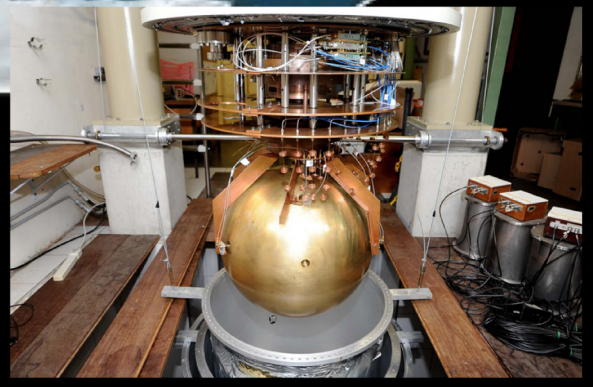



Livingston
Hanford

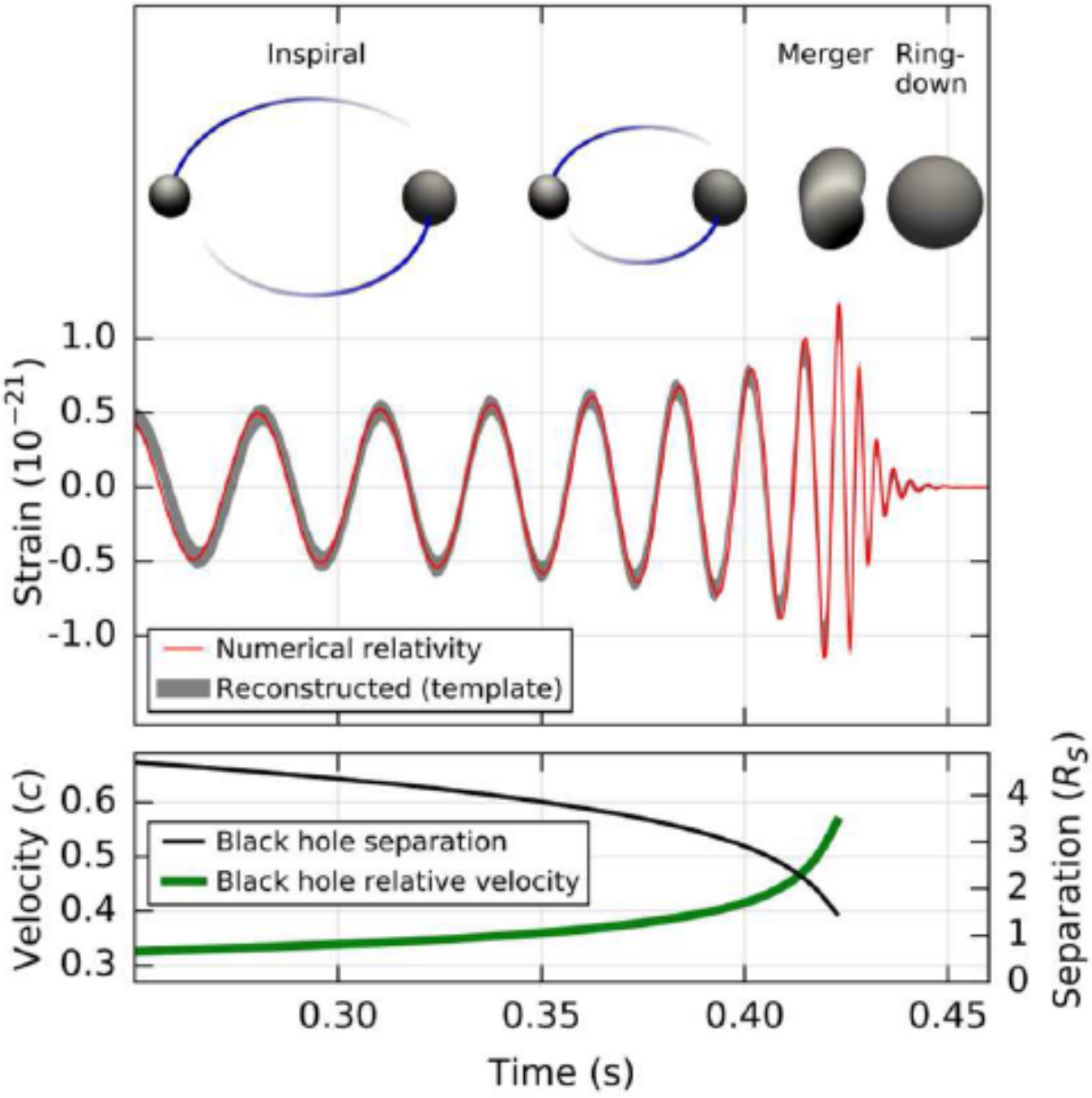
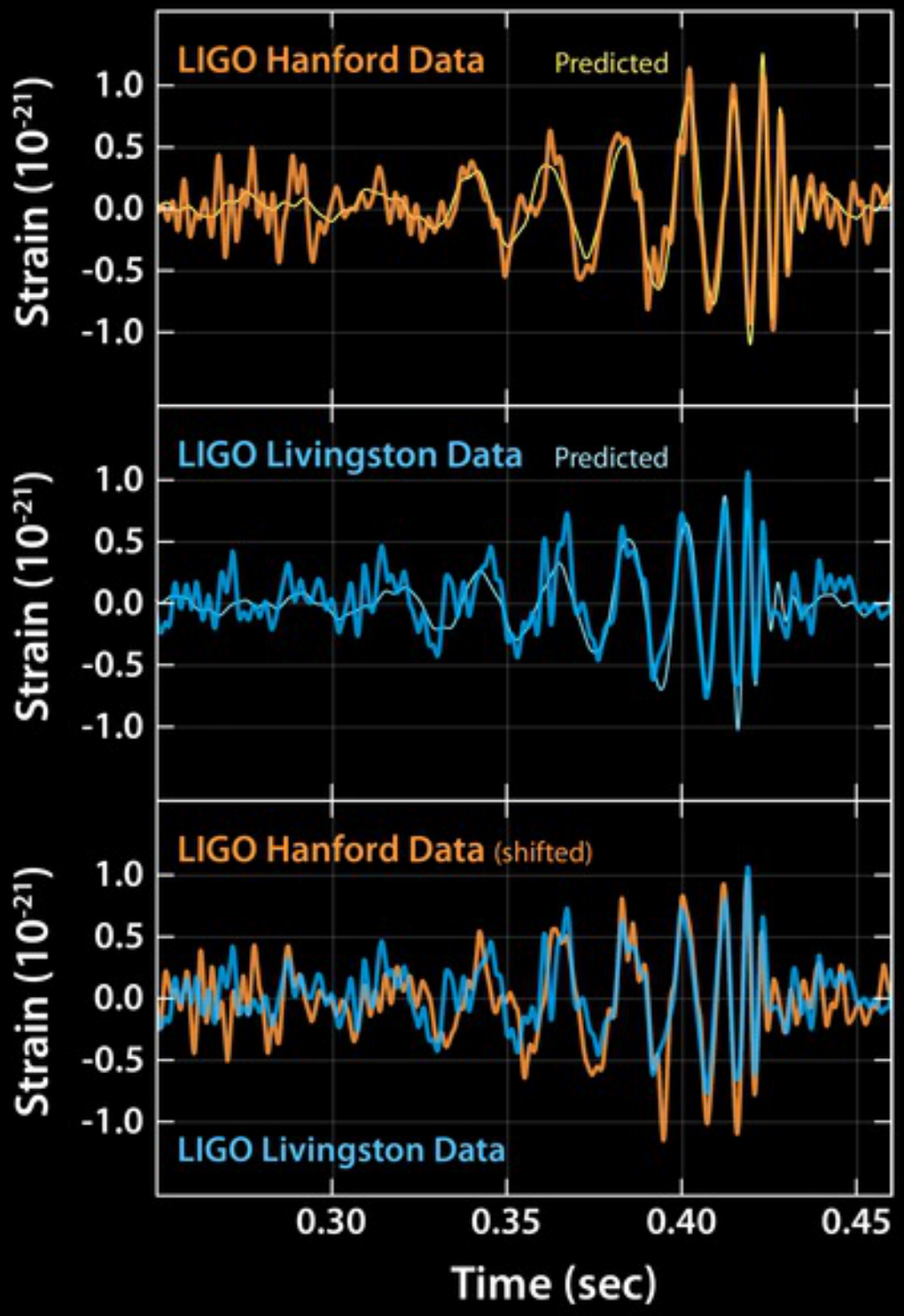


***Figure 3**. Some key results of our analysis of GW150914, comparing the reconstructed gravitational-wave strain (as seen by H1 at Hanford) with the predictions of the best-matching waveform computed from general relativity, over the three stages of the event: inspiral, merger and ringdown. Also shown are the separation and velocity of the black holes, and how they change as the merger event unfolds.*


LIGO Hanford Data
Predicted
LIGO Livingston Data
Predicted
LIGO Hanford Data (shifted)
LIGO Livingston Data
Strain (10-21)
1.0
0.5
0.0
-0.5
-1.0
0.30
0.35
0.40
0.45
Time (sec)

FACT SHEET

observed by	LIGO L1, H1
source type	black hole (BH) binary
date	14 Sept 2015
time	09:50:45 UTC
likely distance	0.75 to 1.9 Gly 230 to 570 Mpc
redshift	0.054 to 0.136
signal-to-noise ratio	24
false alarm prob.	< 1 in 5 million
false alarm rate	< 1 in 200,000 yr
signal arrival time delay	arrived in L1 7 ms before H1
likely sky position	Southern Hemisphere

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(Andrômeda: 2 Mly)

FACT SHEET

Source Masses	M⊙
total mass	60 to 70
primary BH	32 to 41
secondary BH	25 to 33
remnant BH	58 to 67
duration from 30 Hz	~ 200 ms
# cycles from 30 Hz	~10
peak displacement of interferometers arms	±0.002 fm
peak GW luminosity	3.6×10^{56} erg s^{-1}
radiated GW energy	2.5-3.5 M⊙

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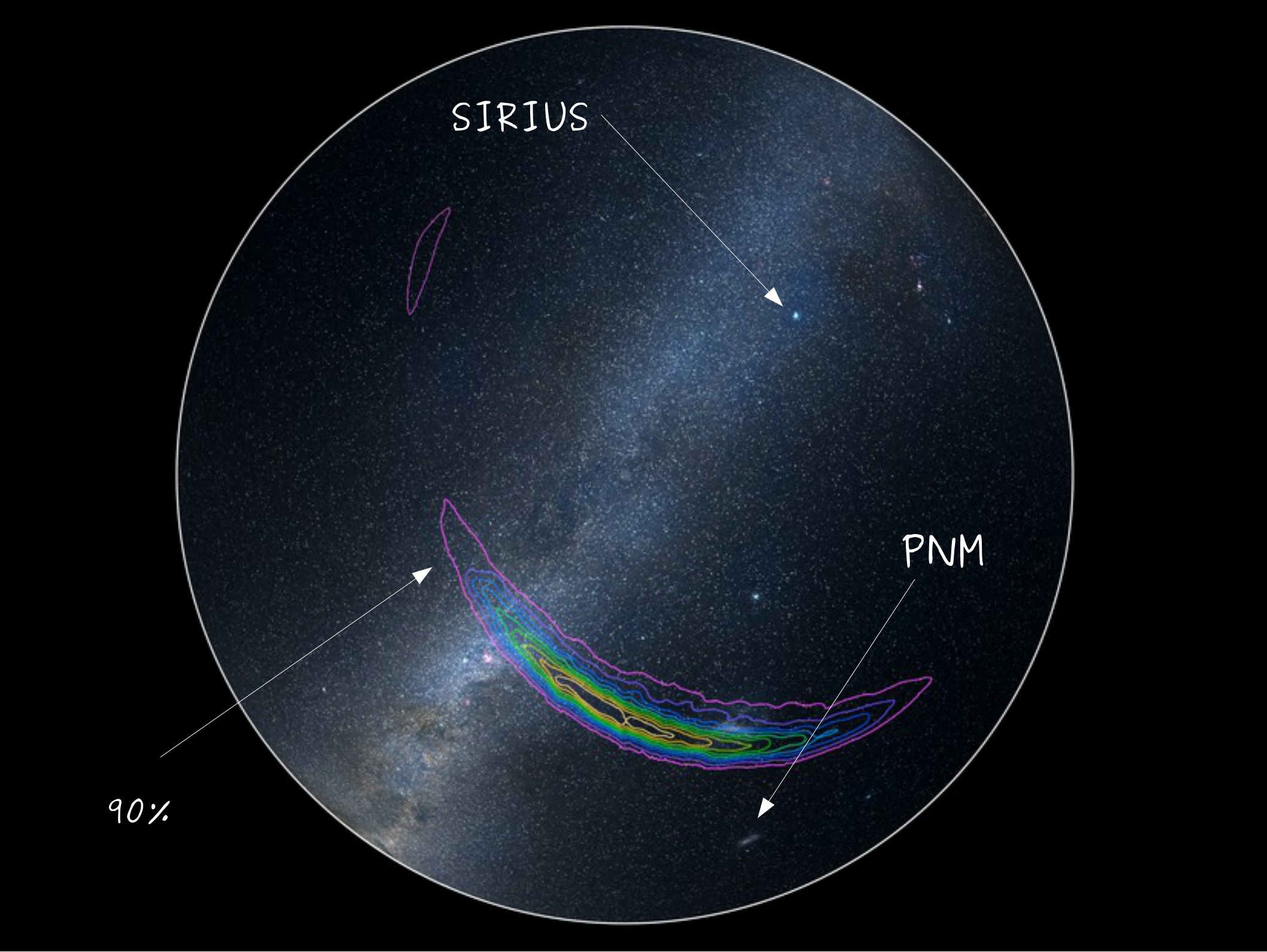
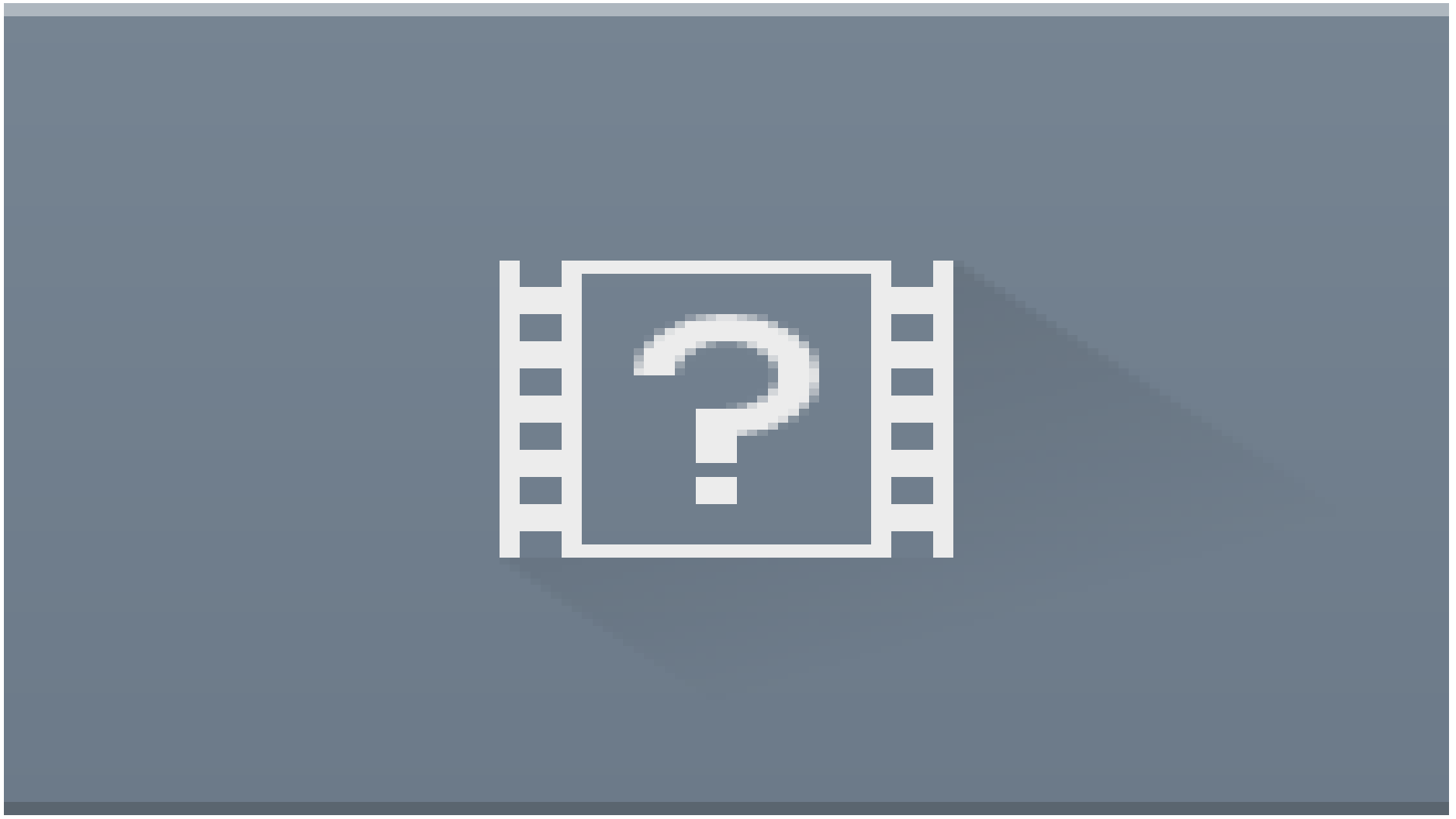
(~50 x todo o universo!)

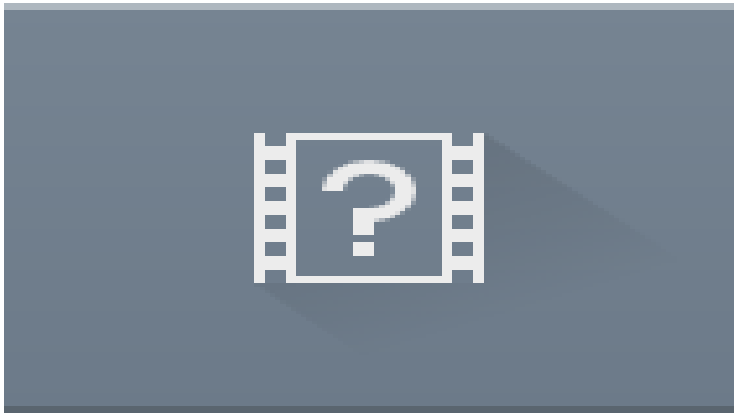
FACT SHEET

consistent with general relativity?	passes all tests performed
graviton mass bound	$< 1.2 \times 10^{-22}$ eV

coalescence rate of binary black holes	2 to 400 $Gpc^{-3}\ yr^{-1}$


SIRIUS
PNM
90%





Bibliografia

- ligo.org
- ligo.caltech.edu
- advancedligo.mit.edu
- livingreviews.org/lrr-2011-5
- PRL 116, 061102 (2016)

Conclusões
(até agora)

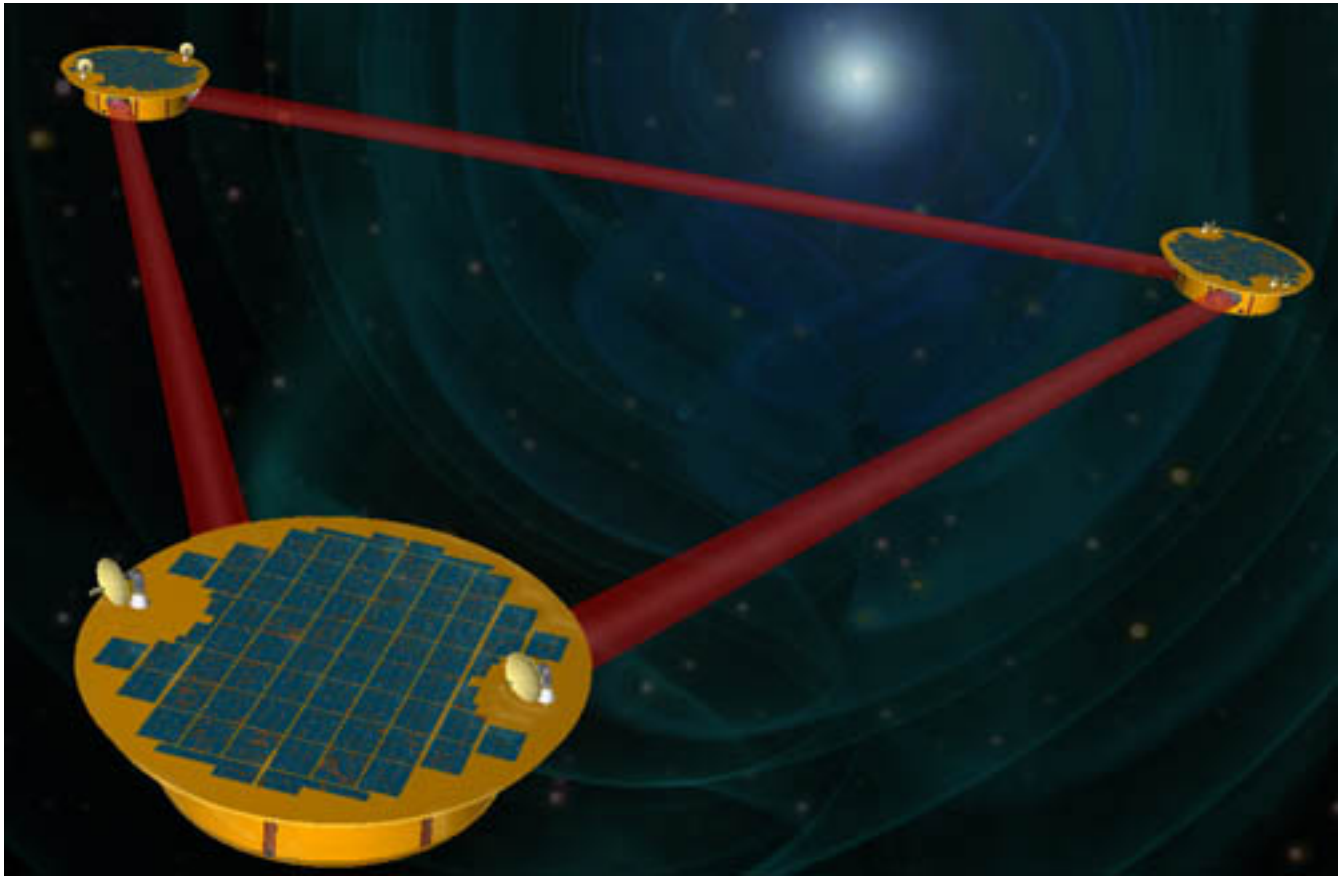
- c = cte: muito forte, mas **é** real!

- A relatividade de Einstein ainda est**á** sendo testada

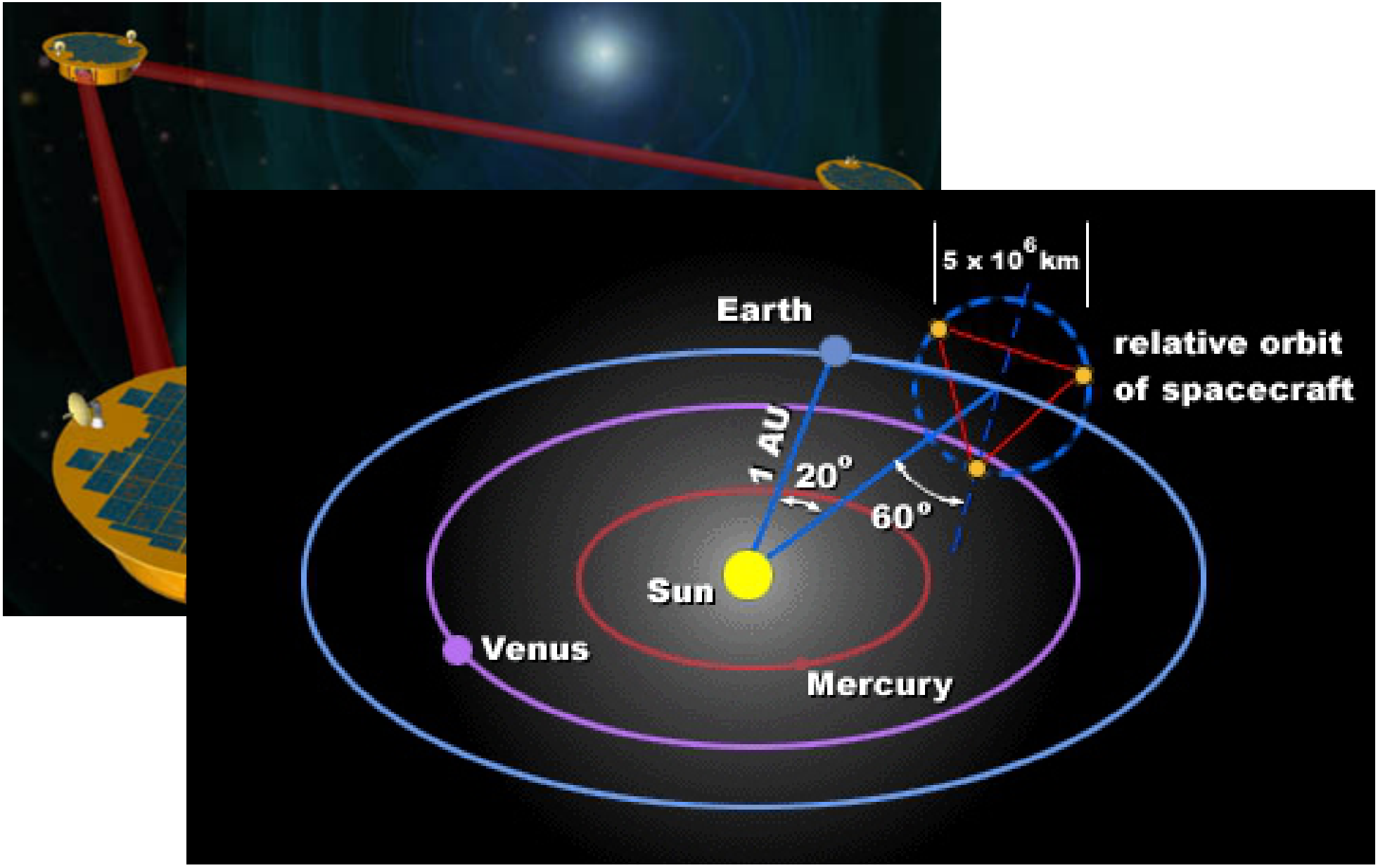
- Ainda h**á** muito o que fazer (vide problemas da matéria e energia escuras)!

~ 96% do universo!

lisa.org



lisa.org


5 x 10^6 km
Earth
relative orbit
of spacecraft
1 AU
20°
60°
Sun
Venus
Mercury

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Astrofísica, Relatividade e COSmologia

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