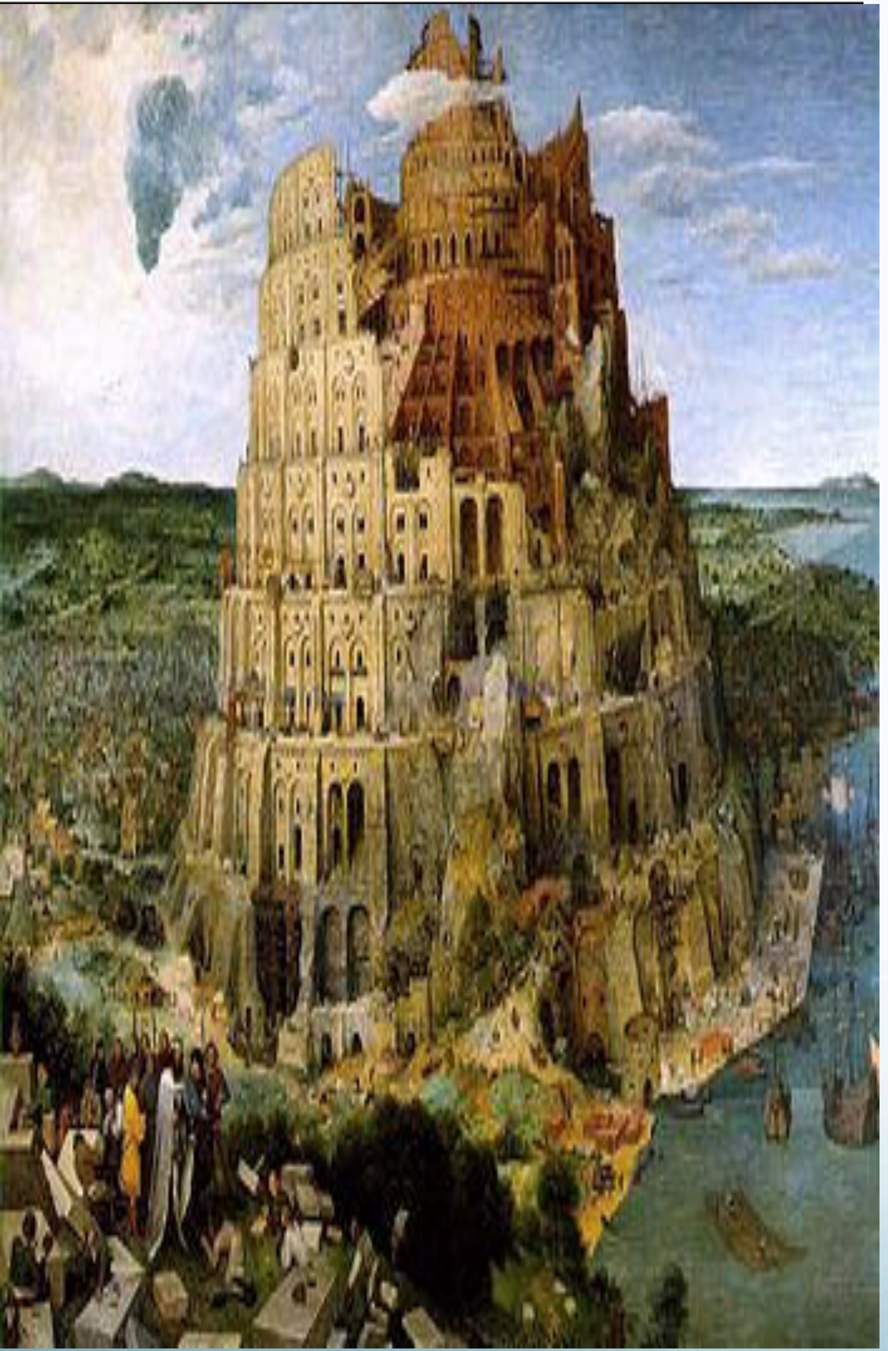
Da torre de Babel a elevadores espaciais e satélites vínculados

Teresa J. Stuchi, IF-UFRJ

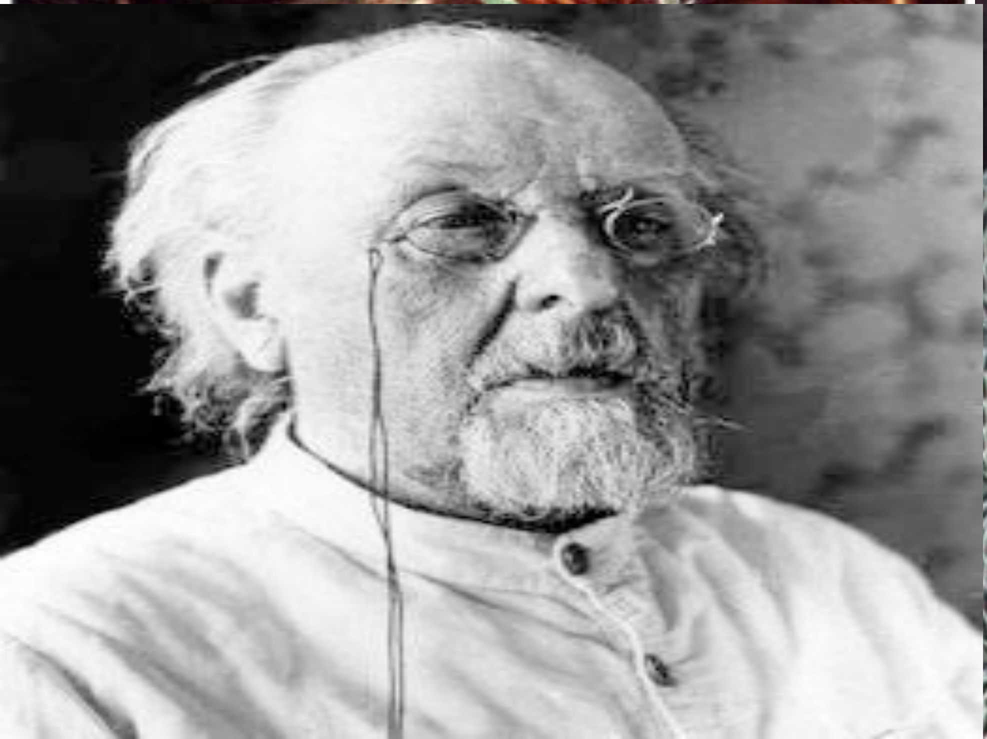
IF-Junho/2015



Space elevators

1. Torre de Babel
 (Moises, Genesis (11:1-9)

2. Escada de Jacob
 (Genesis (28:12))



Space elevators

3. K.E.Tsiolkovski

Speculation about Earth and Sky and on Vesta (1895)

4. Yuri Artsutanov (1960)

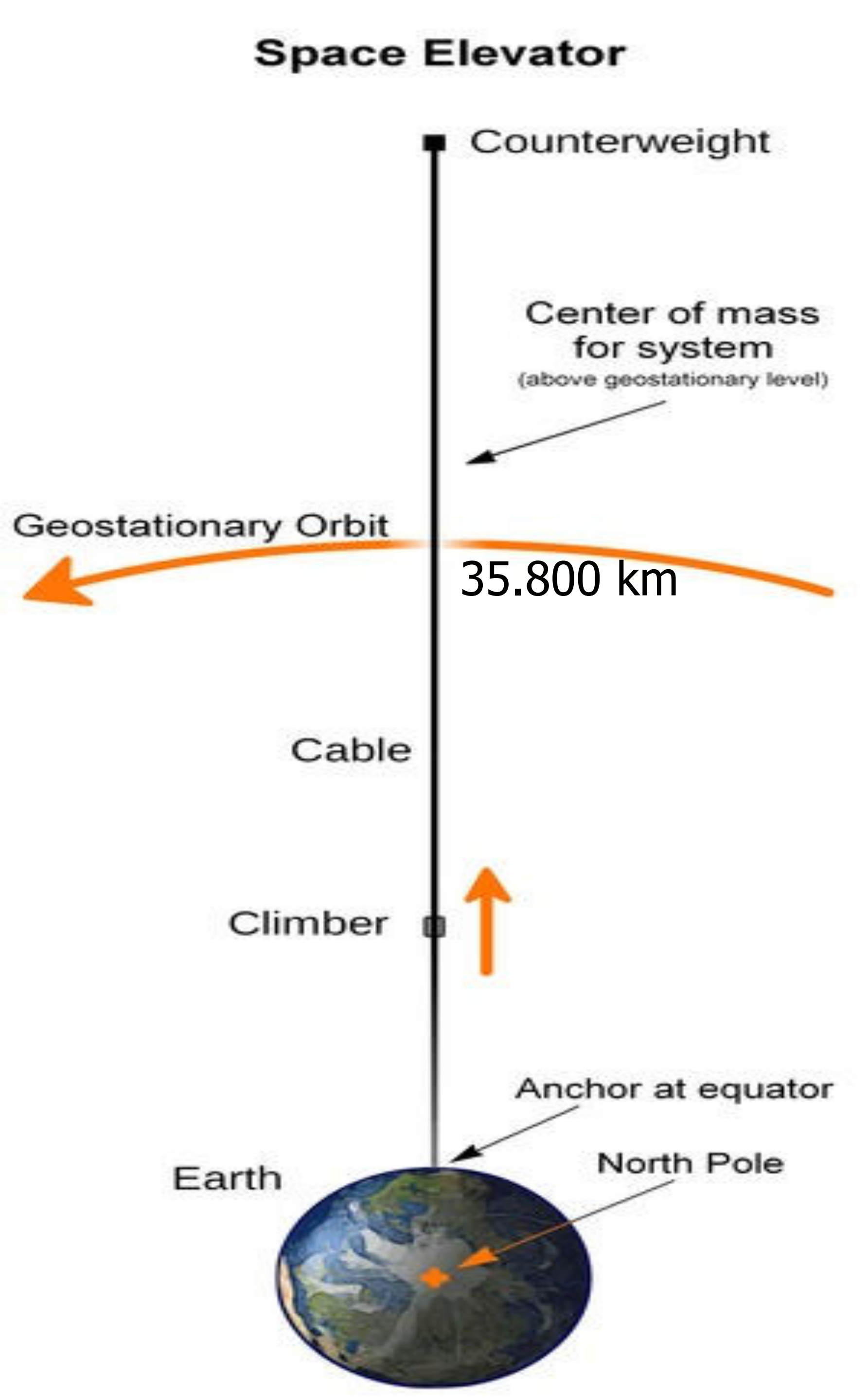
Komosomolskaya Pravda

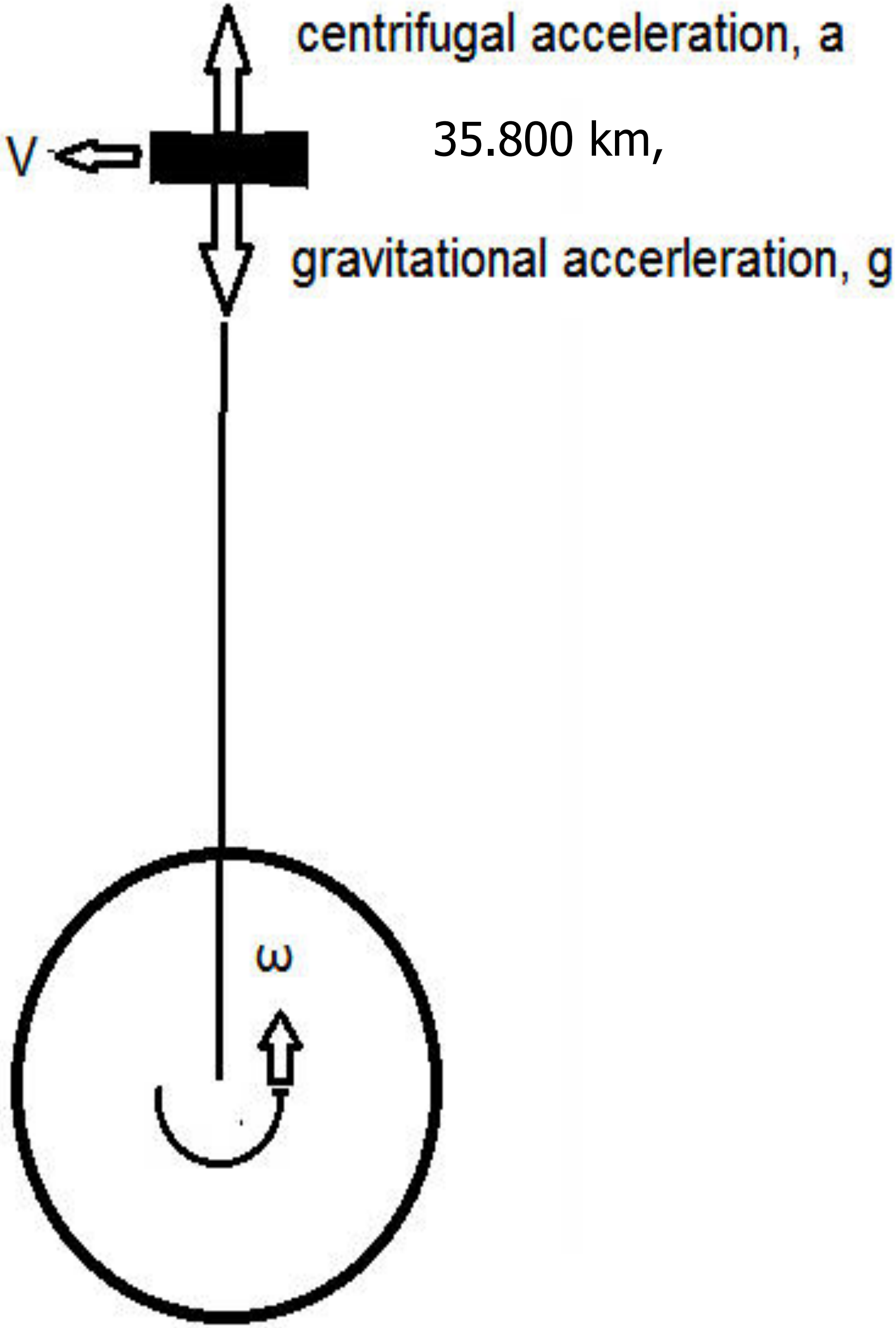
5. Arthur C. Clarke

The fountains of Paradise (tower in Sri Lanka)

Yuri Artsutanov and Jerome Pearson

K.E.Tsiolkovski





GEO: órbita geoestacionária

$$g = GM/r^2$$

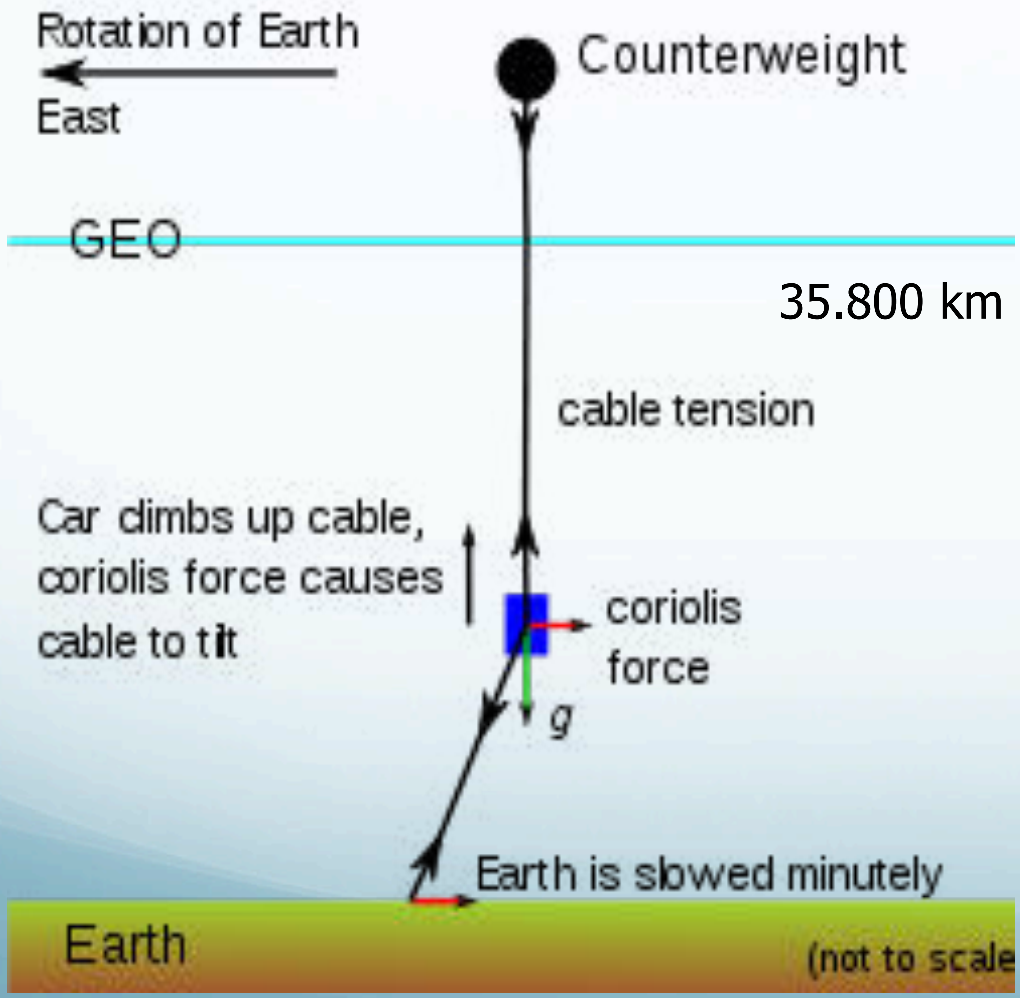
$$a = \omega^2 r$$

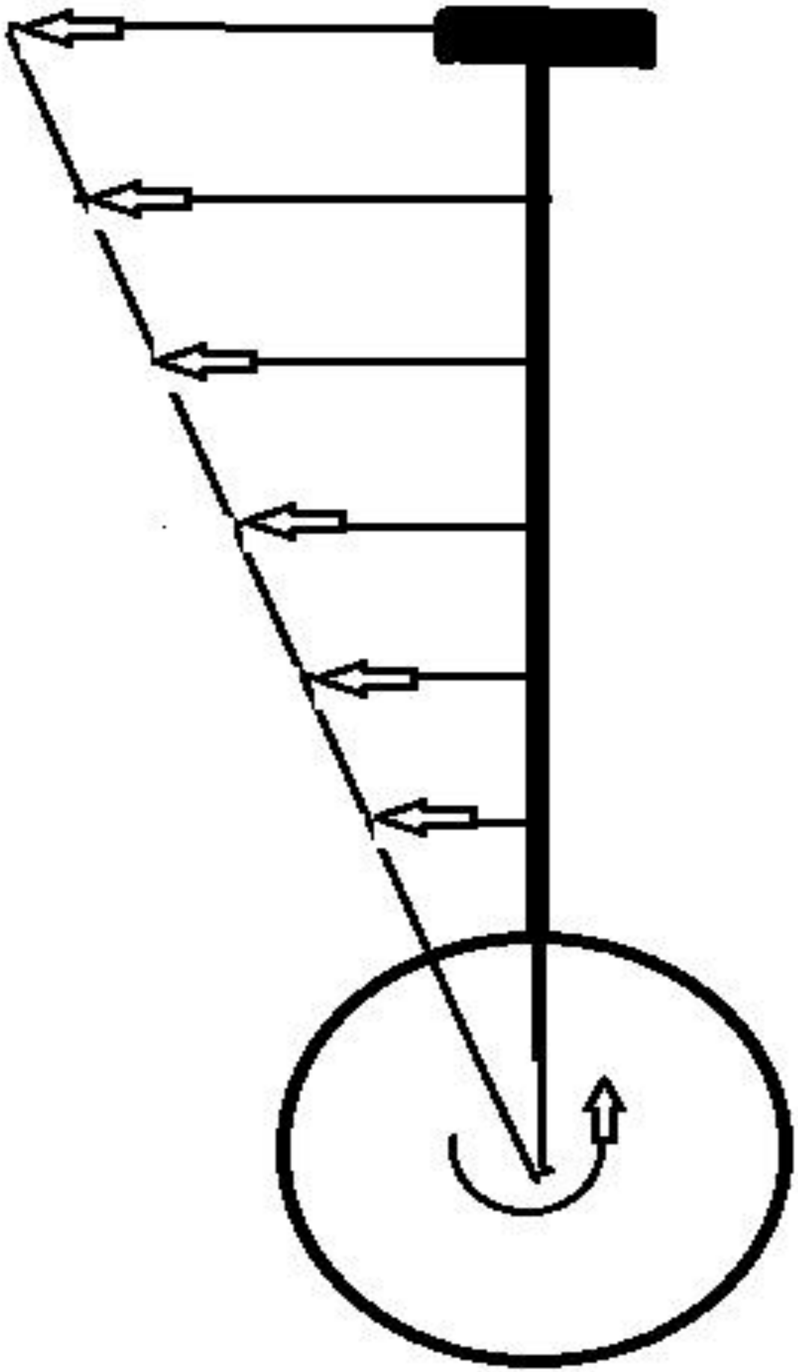
$$g - a = GM/r^2 - \omega^2 r = 0$$

$$r = (GM/\ \omega^2)^{1/3} = R + h$$

Altitude, h = 22,236 miles

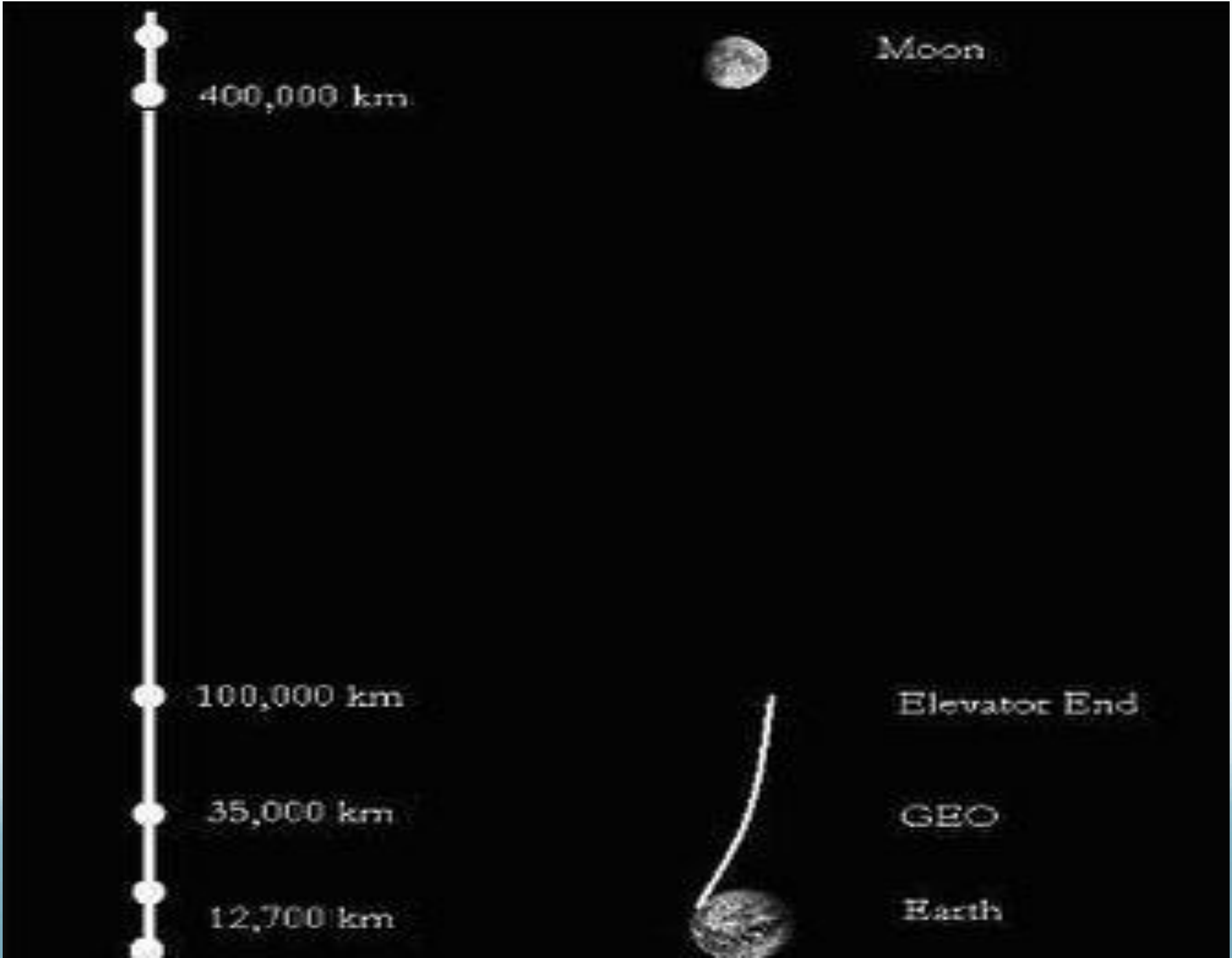
AS THE CAR CLIMBS, THE ELEVATOR TAKES ON A 1 DEGREE LEAN, DUE TO THE TOP OF THE ELEVATOR TRAVELING FASTER THAN THE BOTTOM AROUND THE EARTH (CORIOLIS FORCE). THIS DIAGRAM IS NOT TO SCALE.





VARIAÇÃO DA VELOCIDADE COM A ALTITUDE

The velocity at the top of the tower is
so great (10.93 km/s) that a payload released from there would escape the Earth without rocket propulsion.



Moon
400,000 km
100,000 km
Elevator End
35,000 km
GEO
12,700 km
Earth

CUSTO

TRANSPORTE DE CARGAS PARA O ESPAÇO

- **ATUALMENTE:**

  $500/kg (sp. elev.) vs $20,000/kg (FOGUETE)

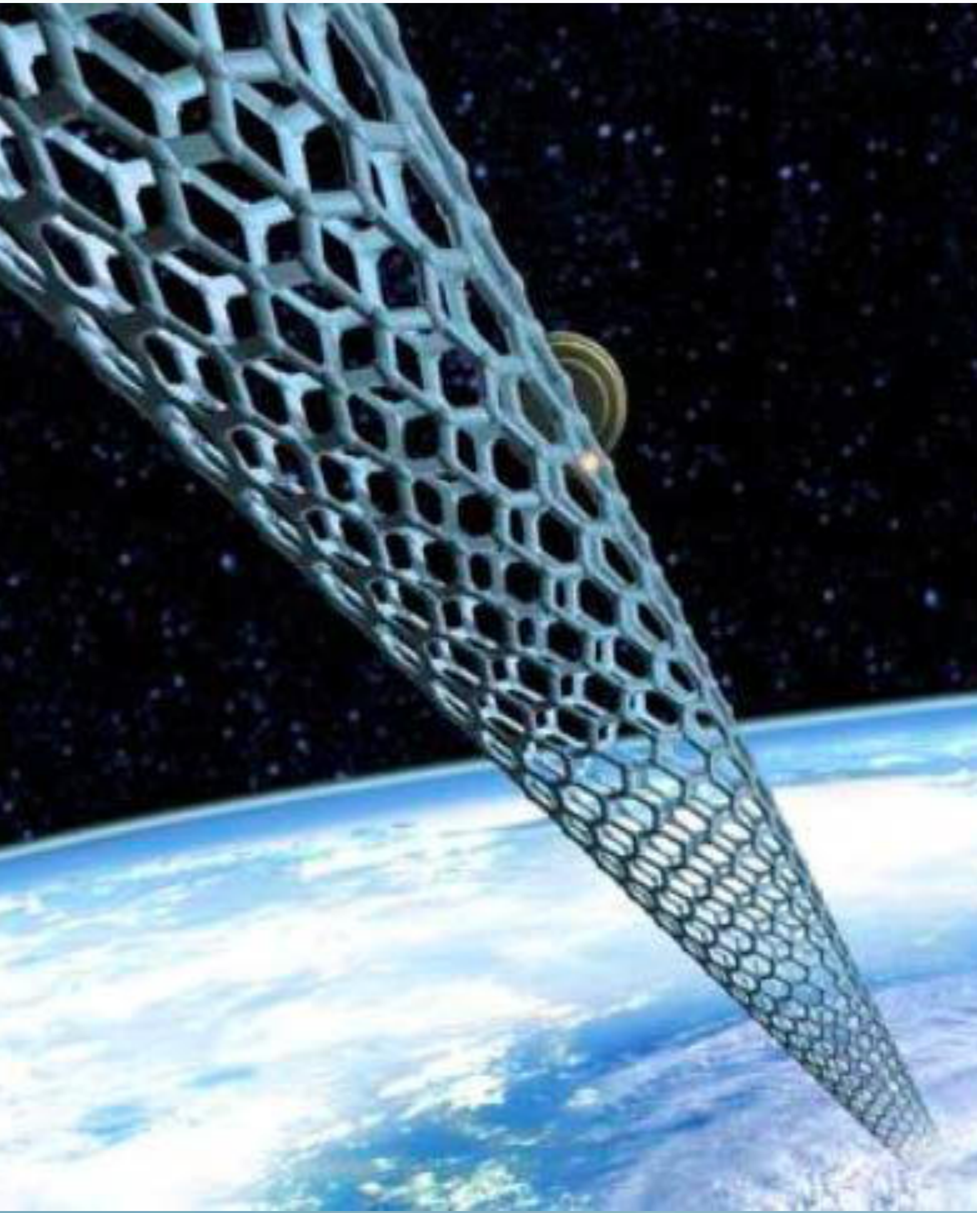
- **FUTURO:**

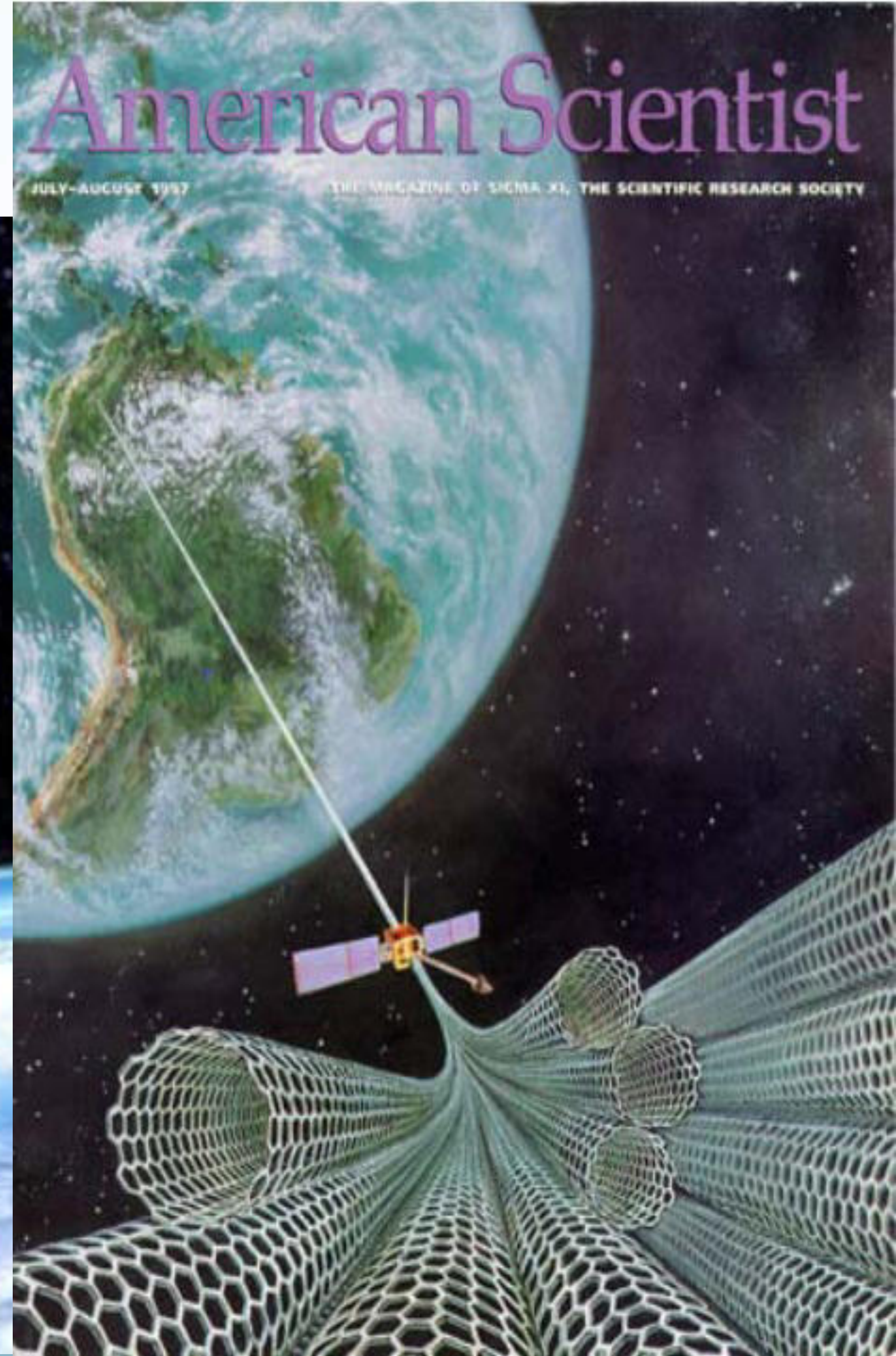
  $100/kg to $8/kg (múltipla)

The Space Elevator will succeed 50 years after everyone has stopped laughing.

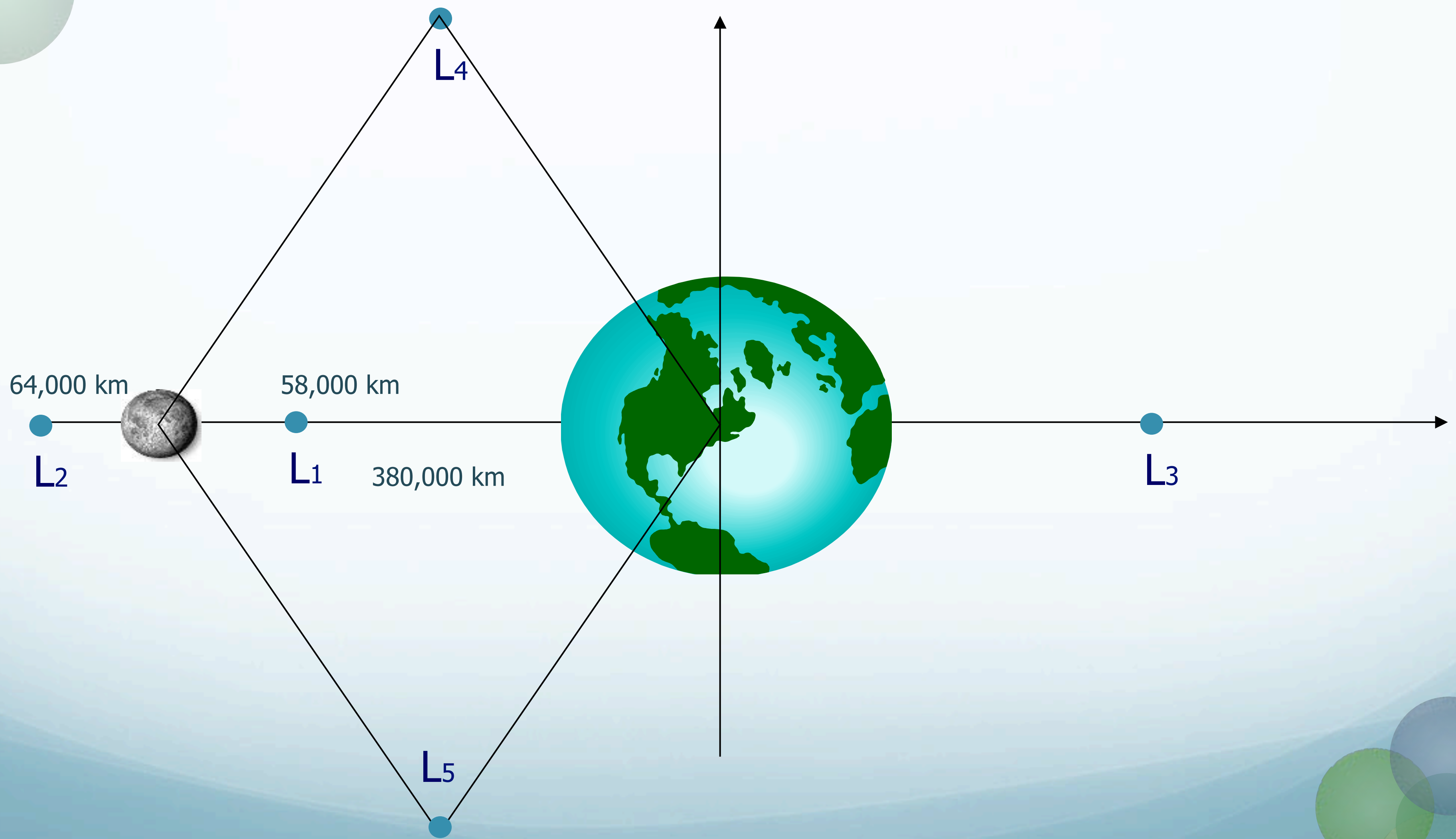
-Arthur C. Clarke

NANO TUBOS DERAM VIDA NOVA AO SONHO DA BABEL

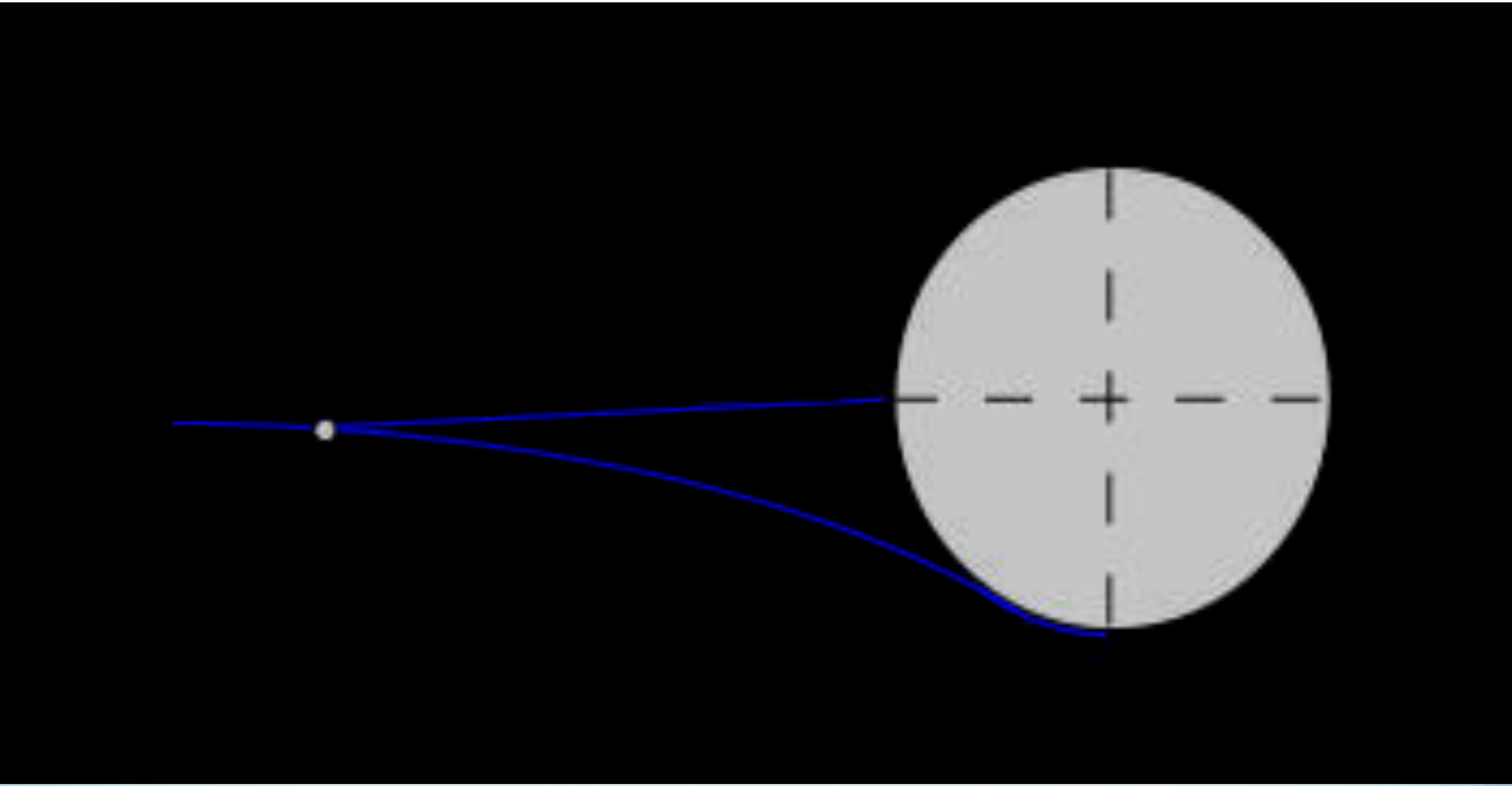




Lagrangian Points
Earth-Moon

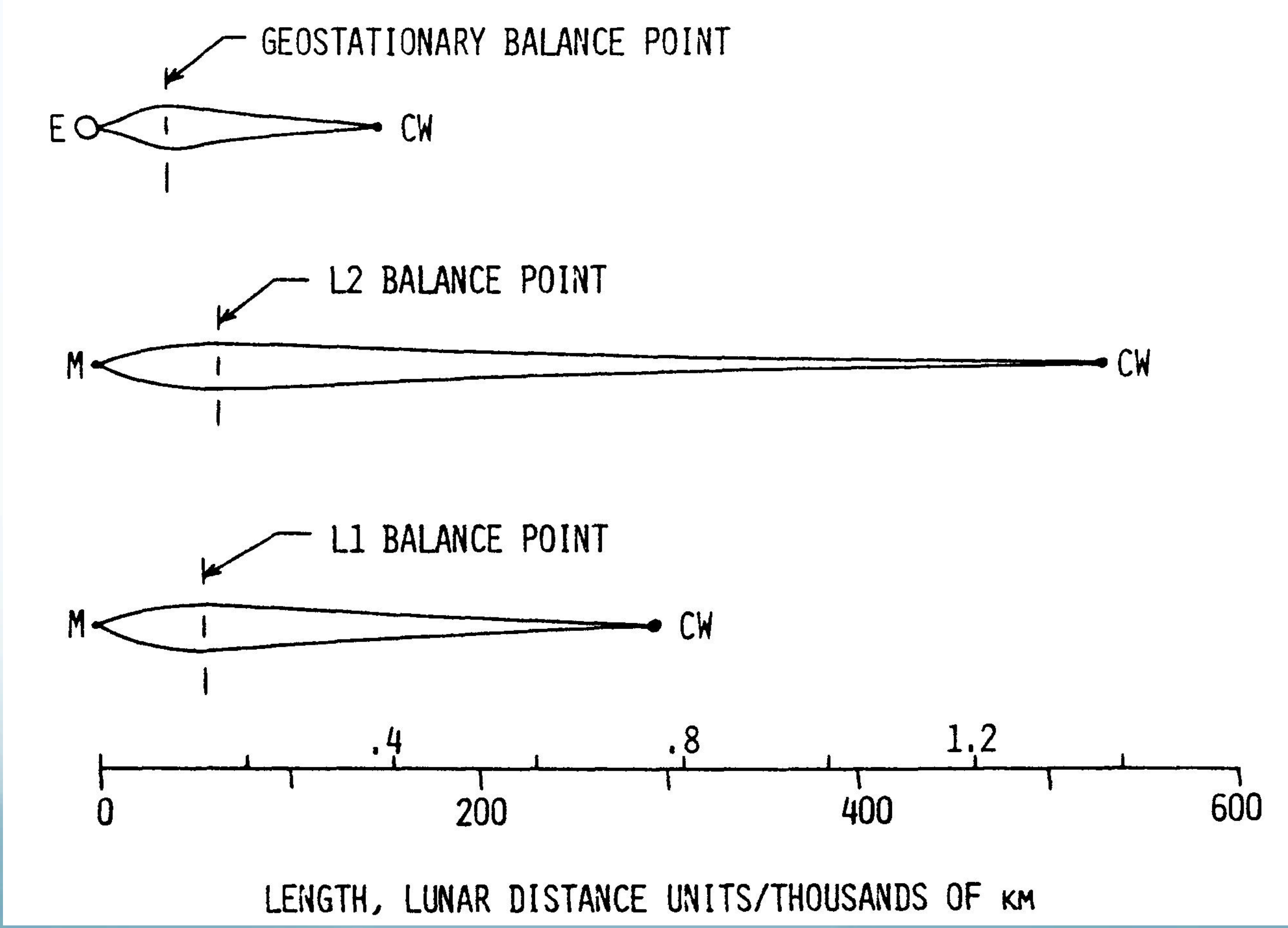


LUNAR SPACE ELEVATOR



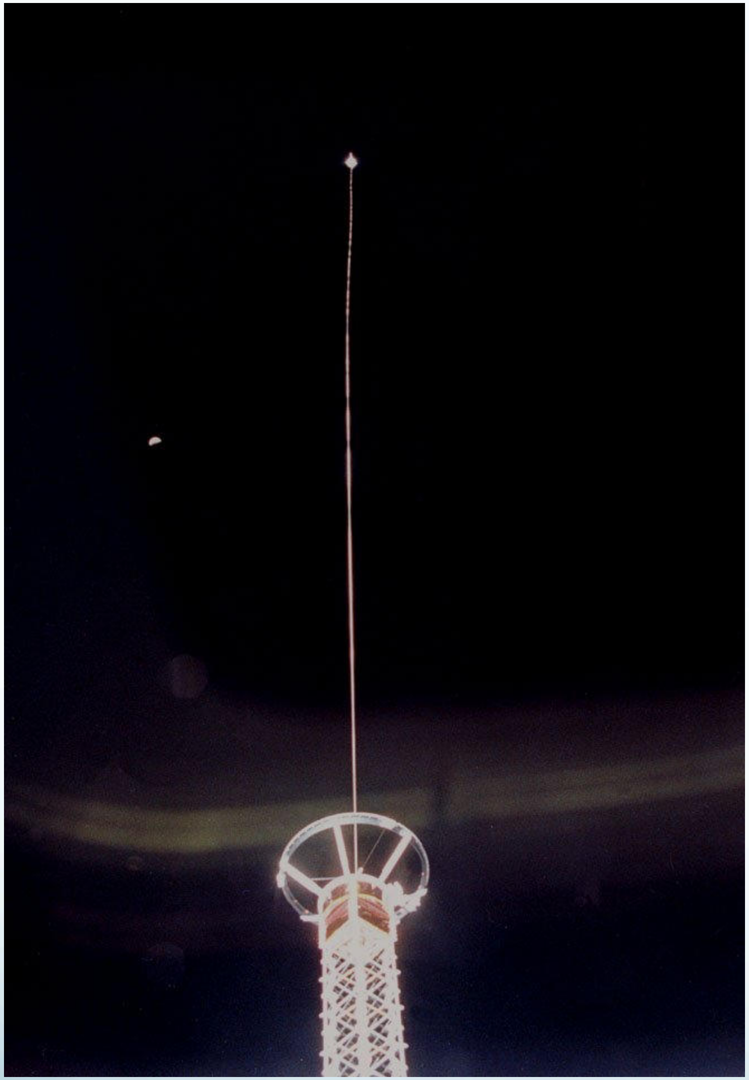
There are two lunar-synchronous points where an elevator could be placed that would be stable: the libration points L_1 and L_2

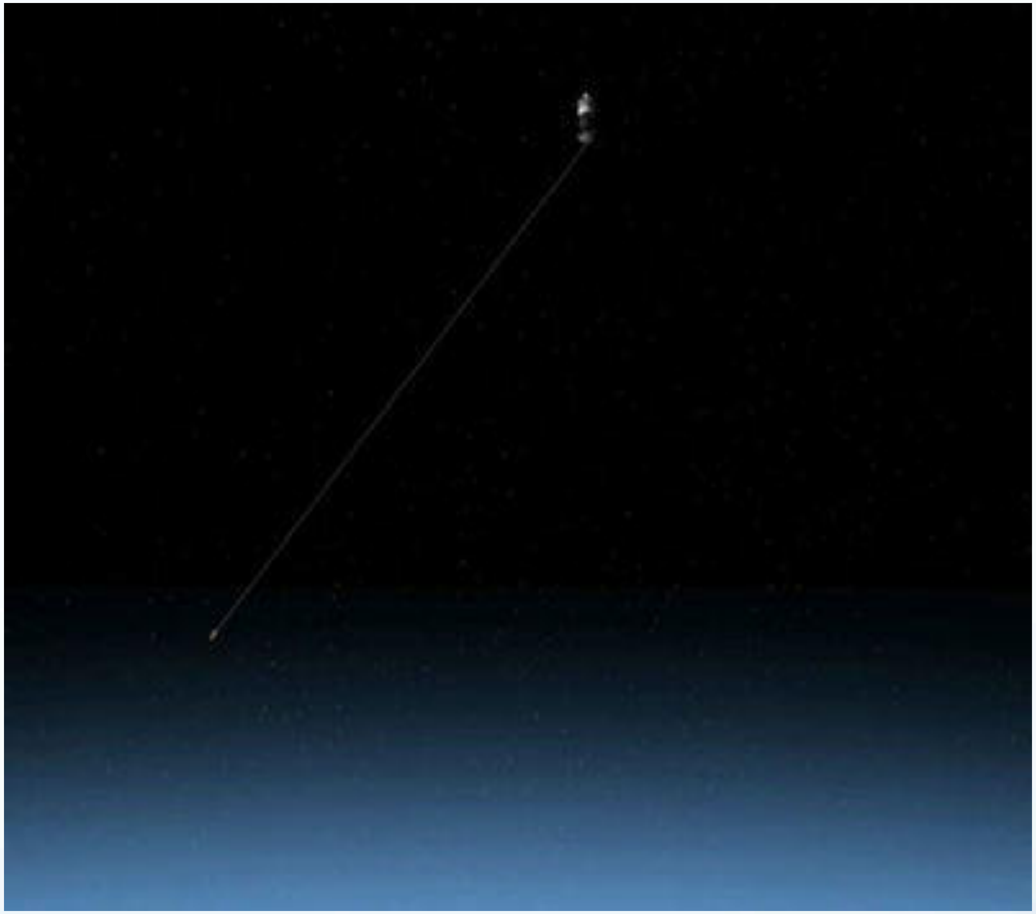
Anchored Lunar Satellites Compared to the Anchored Earth Satellite (Jerome Pearson)



What are tethered satellites?

Artistic representation (NASA)

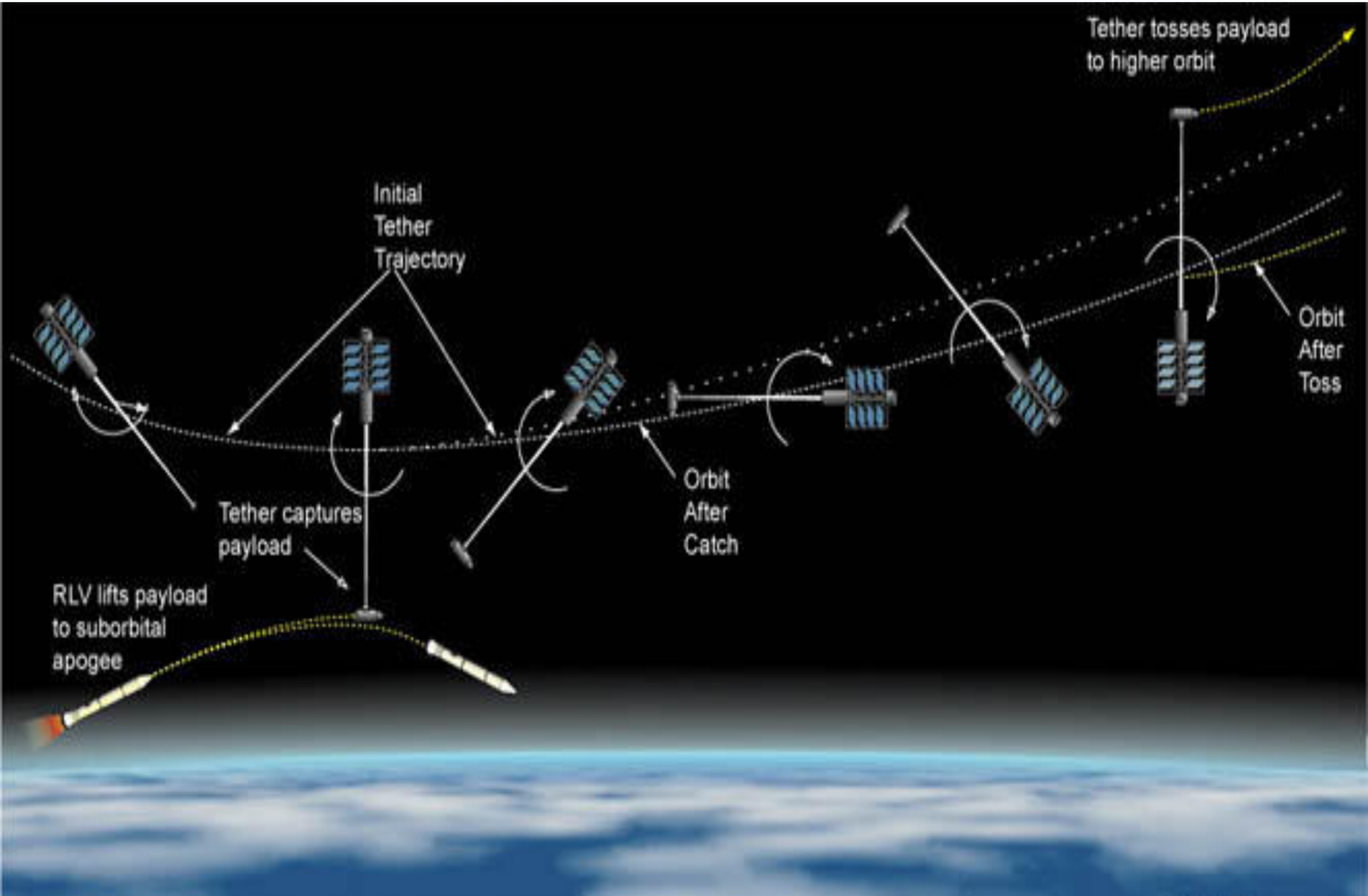




YES2 (2007) - ESA

TSS-1 (1992) - NASA

Tether mudando a órbita de satélite

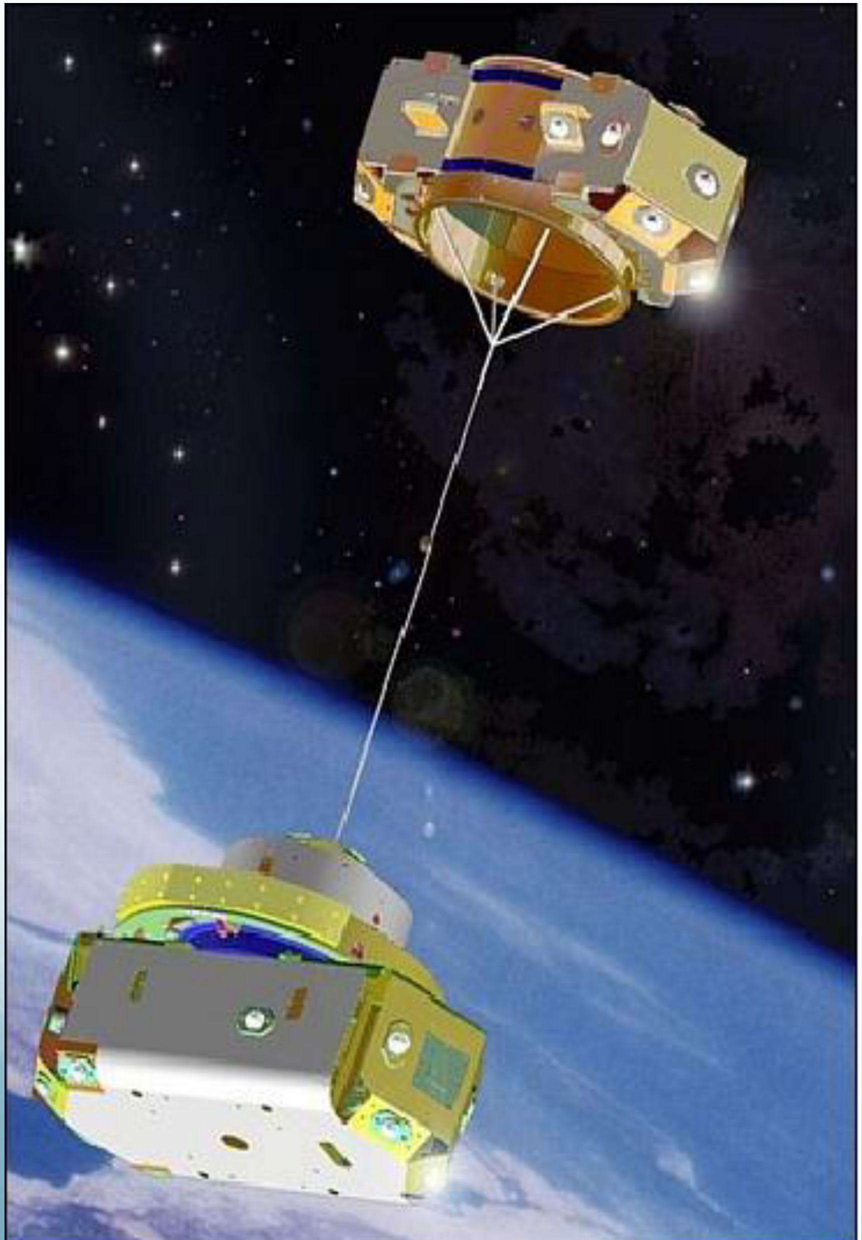


- **Momentum-exchange tethers** (nonconductive tethers representing passive propulsion).

- They allow momentum to be transferred between objects in space, such as two spacecraft (tethers may redistribute momentum of a system from one body to another, but overall momentum is always conserved). The principle is based on the gravity gradient force.

Two objects, separated by a distance but tied together by a tether, are “pulled” apart by the gravity gradient force [this causes vertical (radial) alignment between the two objects]. Due to irregularities in the central body's gravitational field, the nearly radially aligned tether system actually librates, or oscillates, in a pendulum-like motion, about the system's center of mass. This swinging motion may be used to raise or lower the orbit of a tandem system without using any propellant.

TIPS



- A bolo tether rotates end-over-end in the orbit plane. This system could propel a payload attached to one end into a different orbit. The bolo could conceivably catch a payload.

- A stationary tether refers to two end-bodies connected by a tether of constant length. The system may be used to drag the lower end-body payload through the higher atmosphere (for sampling) and simultaneously lowering the system's orbit.

- A tethered system release of an end-body from the remaining end-body and tether causes a momentum gain for the released end-body, resulting in a higher orbit for the released end-body orbit and a lower orbit for the remaining end-body and tether.

Summary: a tether at L2

- The analytical model for the dynamics of

  a tether in the vicinity of collinear points

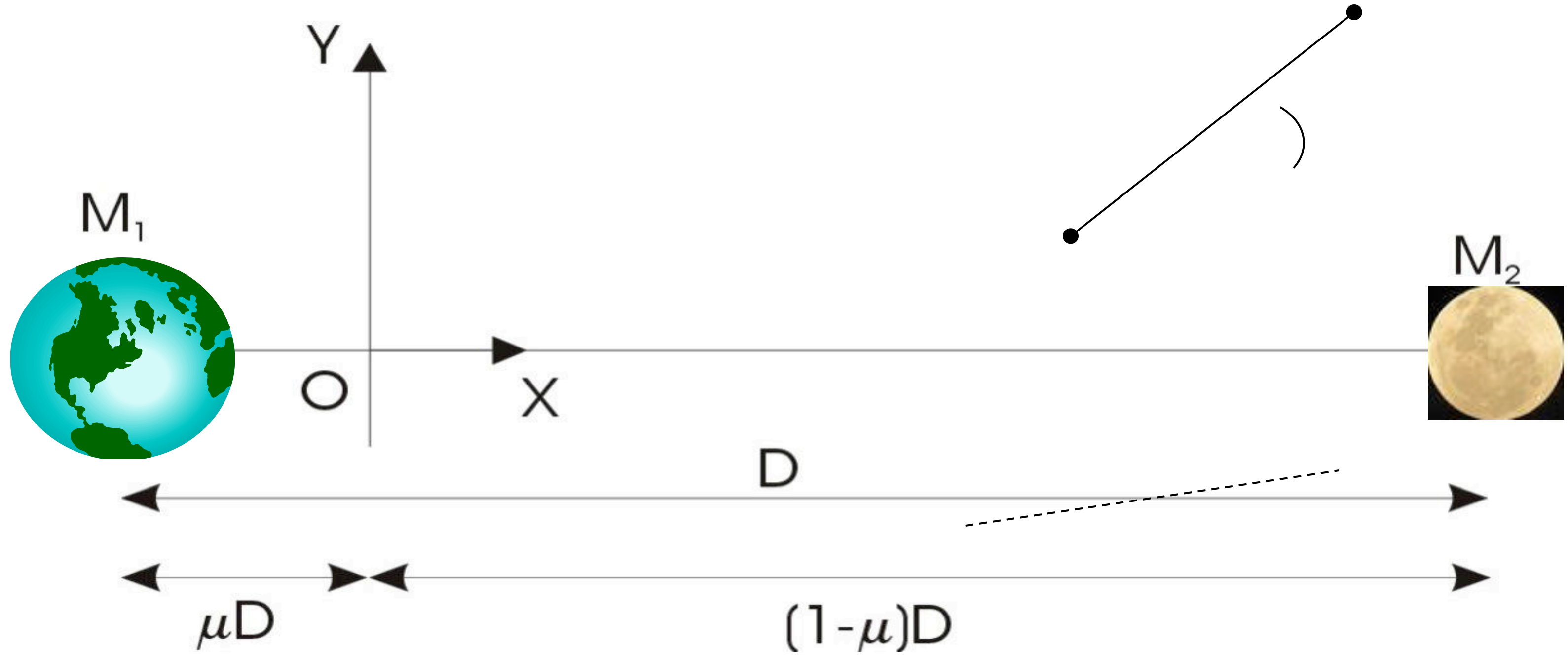
- Determination of the equilibrium points

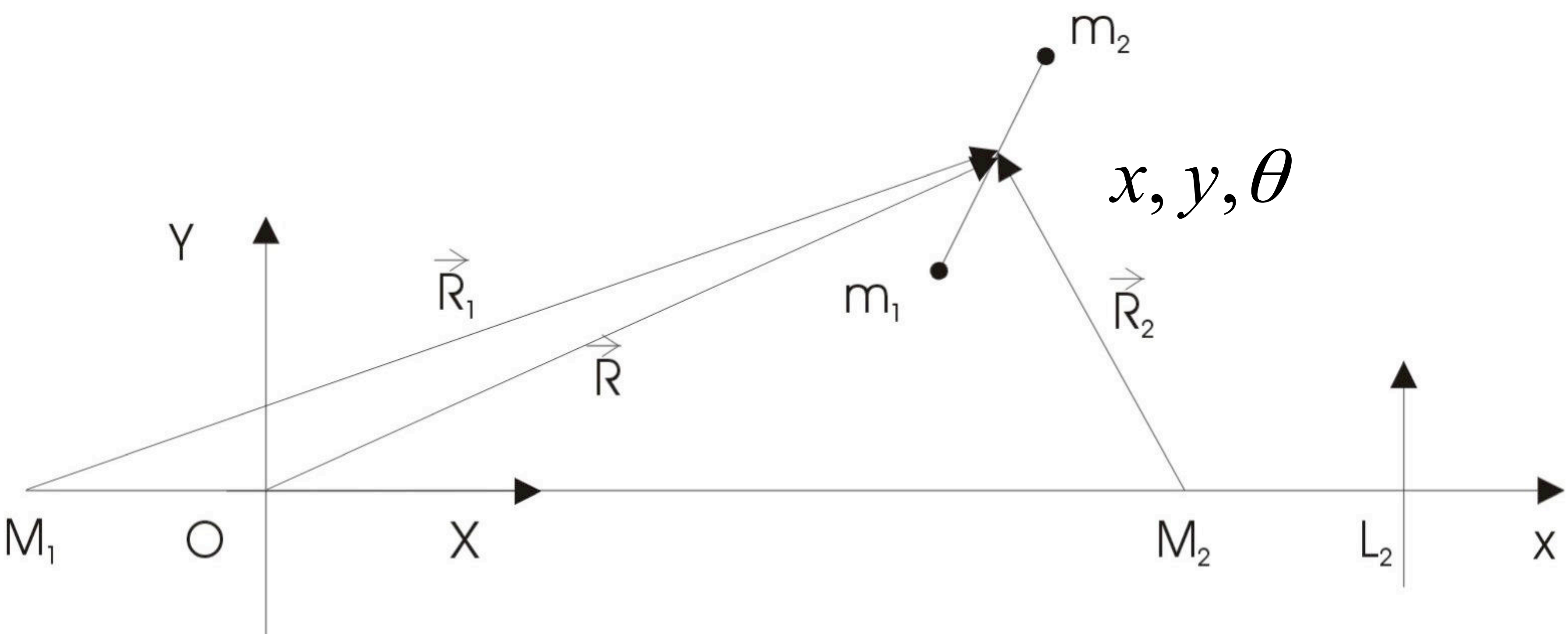
- Lyapunov periodic orbits in the vicinity of the equilibrium points

Application to the Earth-Moon L2

Modelling the tether

- Primary bodies as point masses on circular orbits around the CM
- Cable length much smaller than the distance between the primary bodies
- Planar motion
- Ideal rigid cable


Y
M1
M2
O
X
D
μD
(1-μ)D

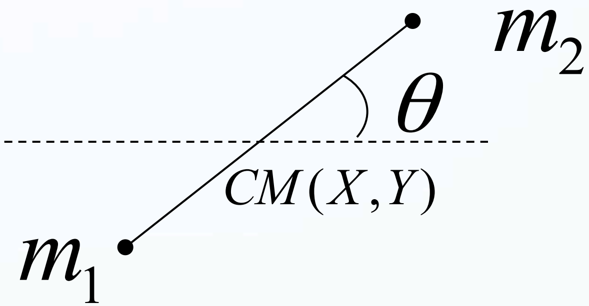

m2
x, y, θ
Y
R1
m1
R2
R
M1
O
X
M2
L2
x

System description

- Synodic system with origin in L2 of the RTBP
- Equations of Euler-Lagrange

generalized coordinates :
(holonomic constraints)

$$x, y, \theta$$



$$L(q,\dot{q}) = T(q,\dot{q}) - V(q)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$$

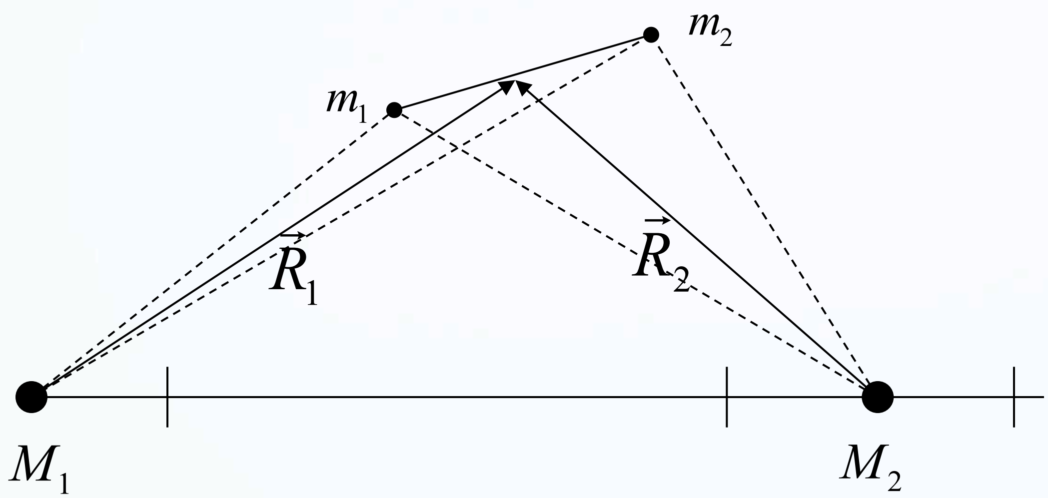
Kinetic energy as a function of the generalized coordinates

$$T = \frac{1}{2}(m_1 + m_2)\{\underbrace{\dot{x}^2 + \dot{y}^2}_{\text{(quadratic)}} \underbrace{- 2n\dot{x}(Y_o + y) + 2n\dot{y}(X_o + x)}_{\text{(Coriolis)}} +$$

$$+ \underbrace{n^2[(X_o + x)^2 + (Y_o + y)^2]}_{\text{(centrifugal)}} +$$

$$+ \underbrace{\beta_1\beta_2[(n + \dot{\theta})^2 l^2 + \dot{l}^2]}_{\text{(rotation/constrain)}}\}$$

Potential energy - V



$$d_1 + d_2 = l$$

$$\frac{1}{\left|\vec{R}_1 - d_1\vec{t}\right|} = \frac{1}{R_1}\left\{1 + \frac{d_1}{R_1}\left(\vec{u}_1.\vec{t}\right) + \left(\frac{d_1}{R_1}\right)^2 \cdot \frac{1}{2}\left(3\left(\vec{u}_1.\vec{t}\right)^2 - 1\right) + \left(\frac{d_1}{R_1}\right)^3 \cdot \frac{1}{2}\left[5\left(\vec{u}_1.\vec{t}\right)^3 - 3\left(\vec{u}_1.\vec{t}\right)\right] + \right.$$

$$V = -\frac{GM_1m_1}{\left|\vec{R}_1 - d_1\vec{t}\right|} - \frac{GM_2m_1}{\left|\vec{R}_2 - d_1\vec{t}\right|} - \frac{GM_1m_2}{\left|\vec{R}_1 + d_2\vec{t}\right|} - \frac{GM_2m_2}{\left|R_2 + d_2\vec{t}\right|} - \frac{GM_1M_2}{D} - \frac{Gm_1m_2}{l}$$

Lagrangian function

$$L = \frac{1}{2}(m_1 + m_2)\left\{ \dot{x}^2 + \dot{y}^2 - 2n\dot{x}(Y_o + y) + 2n\dot{y}(X_o + x) + n^2[(X_o + x)^2 + (Y_o + y)^2] + \beta_1\beta_2[(n+\dot{\theta})^2 l^2 + \dot{l}^2] \right\} +$$

$$G(m_1 + m_2)\left\{ M_1\left[\frac{1}{[(X_0 + D_1 + x)^2 + y^2]^{1/2}} + \frac{1}{2}\frac{\beta_1\beta_2 l^2(3\cos^2\theta - 1)}{[(X_0 + D_1 + x)^2 + y^2]^{3/2}} + \frac{1}{2}\frac{\beta_1\beta_2(\beta_2 - \beta_1)l^3(5\cos^3\theta - 3\cos\theta)}{[(X_0 + D_1 + x)^2 + y^2]^2} + \frac{1}{8}\frac{\beta_1\beta_2(1 - 3\beta_1\beta_2)l^4(35\cos^4\theta - 30\cos^2\theta + 3)}{[(X_0 + D_1 + x)^2 + y^2]^{5/2}} \right] \right.$$

$$\left. M_2\left[\frac{1}{[(X_0 - D_2 + x)^2 + y^2]^{1/2}} + \frac{1}{2}\frac{\beta_1\beta_2 l^2(3\cos^2\theta - 1)}{[(X_0 - D_2 + x)^2 + y^2]^{3/2}} + \frac{1}{2}\frac{\beta_1\beta_2(\beta_2 - \beta_1)l^3(5\cos^3\theta - 3\cos\theta)}{[(X_0 - D_2 + x)^2 + y^2]^2} + \frac{1}{8}\frac{\beta_1\beta_2(1 - 3\beta_1\beta_2)l^4(35\cos^4\theta - 30\cos^2\theta + 3)}{[(X_0 - D_2 + x)^2 + y^2]^{5/2}} \right] \right\}$$

Expansion up to fourth order

Equations of motion

dimensionless coordinates : $\hat{x} = x/D \quad \hat{y} = y/D \quad \hat{l} = l/D \quad \tau = nt$

$$\begin{cases} \hat{x}'' = +2\hat{y}' - \left(\dfrac{1-\mu}{\hat{R}_1^3}\Lambda_1^{l,\theta} + \dfrac{\mu}{\hat{R}_2^3}\Lambda_2^{l,\theta} - 1\right)(\hat{X}_0 + \hat{x}) - \mu(1-\mu)\left(\dfrac{\Lambda_1^{l,\theta}}{\hat{R}_1^3} - \dfrac{\Lambda_2^{l,\theta}}{\hat{R}_2^3}\right) \\ \hat{y}'' = -2\hat{x}' - \left(\dfrac{1-\mu}{\hat{R}_1^3}\Lambda_1^{l,\theta} + \dfrac{\mu}{\hat{R}_2^3}\Lambda_2^{l,\theta} - 1\right)\hat{y} \\ \theta'' = -2\left(\dfrac{\hat{l}'}{\hat{l}}\right)(\theta' + 1) - \dfrac{1-\mu}{\hat{R}_1^3}\Omega_1^{l,\theta} - \dfrac{\mu}{R_2^3}\Omega_2^{l,\theta} \end{cases}$$

, where:

$$\Lambda_i^{l,\theta} = 1 + \frac{3}{2}\beta_1\beta_2\left(\frac{l}{R_1}\right)^2(3\cos^2\theta - 1) + 2\beta_1\beta_2(\beta_2 - \beta_1)\left(\frac{l}{R_1}\right)^3(5\cos^3\theta - 3\cos\theta) + \frac{5}{8}\beta_1\beta_2(1 - 3\beta_1\beta_2)\left(\frac{l}{R_1}\right)^4\left(35\cos^4\theta - 30\cos^2\theta + 3\right)$$

$$\Omega_i^{l,\theta} = \frac{3}{2}sen(2\theta) - \frac{3}{2}(\beta_2 - \beta_1)\left(\frac{l}{R_i}\right)(1 - 5\cos^2\theta)sen\theta - \frac{5}{2}(1 - 3\beta_1\beta_2)\left(\frac{l}{R_i}\right)^2(3 - 7\cos^2\theta)\cos\theta sen\theta$$

$$i \in \{1,2\}$$

Constant of motion

$$E = \sum_{i=1}^{n} \dot{q}_i \frac{\partial L}{\partial \dot{q}_i} - L$$

(Jacobian constant)

$$E^* = 2\Omega - (\hat{x}'^2 + \hat{y}'^2) - \beta_1\beta_2\left[(\theta'^2 - 1)\hat{l}^2 - \hat{l}'^2\right]$$

, where:

$$\Omega = \frac{(\hat{X}_0 + \hat{x})^2}{2} + \frac{\hat{y}^2}{2} + \frac{1-\mu}{\hat{R}_1}\Sigma_1 + \frac{\mu}{\hat{R}_2}\Sigma_2$$

$$\Sigma_i = 1 + \frac{1}{2}\beta_1\beta_2\left(\frac{l}{R_i}\right)^2(3\cos^2\theta - 1) + \frac{1}{2}\beta_1\beta_2(\beta_2 - \beta_1)\left(\frac{l}{R_i}\right)^3(5\cos^3\theta - 3\cos\theta) +$$

$$\frac{1}{8}\beta_1\beta_2(1 - 3\beta_1\beta_2)\left(\frac{l}{R_i}\right)^4(35\cos^4\theta - 30\cos^2\theta + 3)$$

$$i \in \{1,2\}$$

If $l = 0 \Longrightarrow$ RTBP

Equations of motion

$$\Lambda_1^{0,0} = \Lambda_2^{0,0} = 1 \quad \Longrightarrow \quad \begin{cases} \hat{X}'' = +2\hat{Y}' - \left(\dfrac{1-\mu}{\hat{R}_1^3} + \dfrac{\mu}{\hat{R}_2^3} - 1\right)\hat{X} - \mu(1-\mu)\left(\dfrac{1}{\hat{R}_1^3} - \dfrac{1}{\hat{R}_2^3}\right) \\ \hat{Y}'' = -2\hat{X}' - \left(\dfrac{1-\mu}{\hat{R}_1^3} + \dfrac{\mu}{\hat{R}_2^3} - 1\right)\hat{y} \end{cases}$$

Jacobian constant

$$\Sigma_1 = \Sigma_2 = 1 \quad \Longrightarrow \quad E^* = C_J = 2\Omega - (\hat{X}'^2 + \hat{Y}'^2)$$

$$\Omega = \frac{\hat{X}^2}{2} + \frac{\hat{Y}^2}{2} + \frac{1-\mu}{\hat{R}_1} + \frac{\mu}{\hat{R}_2}$$

$$X_0 + x \leftrightarrow X$$

Equilibrium equations

Left hand side equals zero and all velocities equal zero

$$\begin{cases} \hat{x}'' = +2\hat{y}' - \left(\frac{1-\mu}{\hat{R}_1^3}\Lambda_1^{l,\theta} + \frac{\mu}{\hat{R}_2^3}\Lambda_2^{l,\theta} - 1\right)(\hat{X}_0 + \hat{x}) - \mu(1-\mu)\left(\frac{\Lambda_1^{l,\theta}}{\hat{R}_1^3} - \frac{\Lambda_2^{l,\theta}}{\hat{R}_2^3}\right) \\ \hat{y}'' = -2\hat{x}' - \left(\frac{1-\mu}{\hat{R}_1^3}\Lambda_1^{l,\theta} + \frac{\mu}{\hat{R}_2^3}\Lambda_2^{l,\theta} - 1\right)\hat{y} \\ \theta'' = -2\left(\frac{\hat{l}'}{\hat{l}}\right)(\theta' + 1) - \frac{1-\mu}{\hat{R}_1^3}\Omega_1^{l,\theta} - \frac{\mu}{R_2^3}\Omega_2^{l,\theta} \end{cases}$$

, where:

$$\Lambda_i^{l,\theta} = 1 + \frac{3}{2}\beta_1\beta_2\left(\frac{l}{R_1}\right)^2(3\cos^2\theta - 1) + 2\beta_1\beta_2(\beta_2 - \beta_1)\left(\frac{l}{R_1}\right)^3(5\cos^3\theta - 3\cos\theta) + \frac{5}{8}\beta_1\beta_2(1-3\beta_1\beta_2)\left(\frac{l}{R_1}\right)^4\left(35\cos^4\theta - 30\cos^2\theta + 3\right)$$

$$\Omega_i^{l,\theta} = \frac{3}{2}sen(2\theta) - \frac{3}{2}(\beta_2 - \beta_1)\left(\frac{l}{R_i}\right)(1 - 5\cos^2\theta)sen\theta - \frac{5}{2}(1-3\beta_1\beta_2)\left(\frac{l}{R_i}\right)^2(3 - 7\cos^2\theta)\cos\theta sen\theta$$

$$i \in \{1,2\}$$

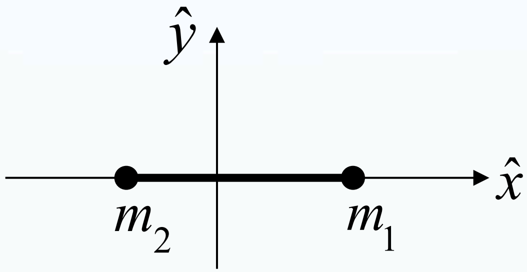
Equilibrium points $(\hat{x}_{eq}, \hat{y}_{eq}, \theta_{eq})$

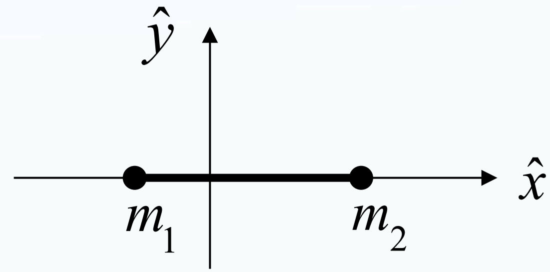
$$\hat{x}_{eq} = \hat{x} \;\Longrightarrow\; \frac{1-\mu}{(\hat{X}_0+\hat{x}+\mu)^2}\Lambda_1^{l,\theta} + \frac{\mu}{(\hat{X}_0+\hat{x}+\mu-1)^2}\Lambda_2^{l,\theta} = \hat{X}_0 + \hat{x} \quad \text{(degree 7)}$$

$$\hat{y}_{eq} = 0$$

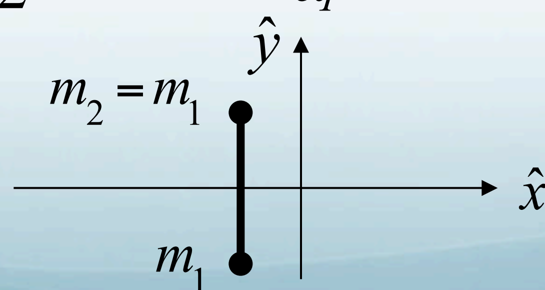
θ_{eq}:

$$\forall m_1, m_2 \;\Longrightarrow\; \theta_{eq} = 0 \; ou \; \theta_{eq} = \pi$$

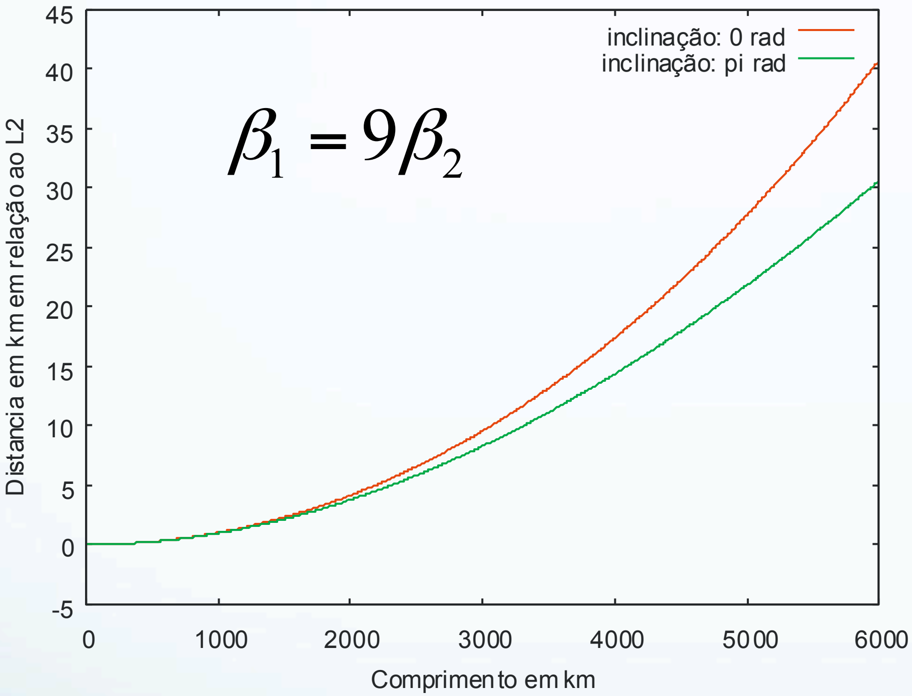


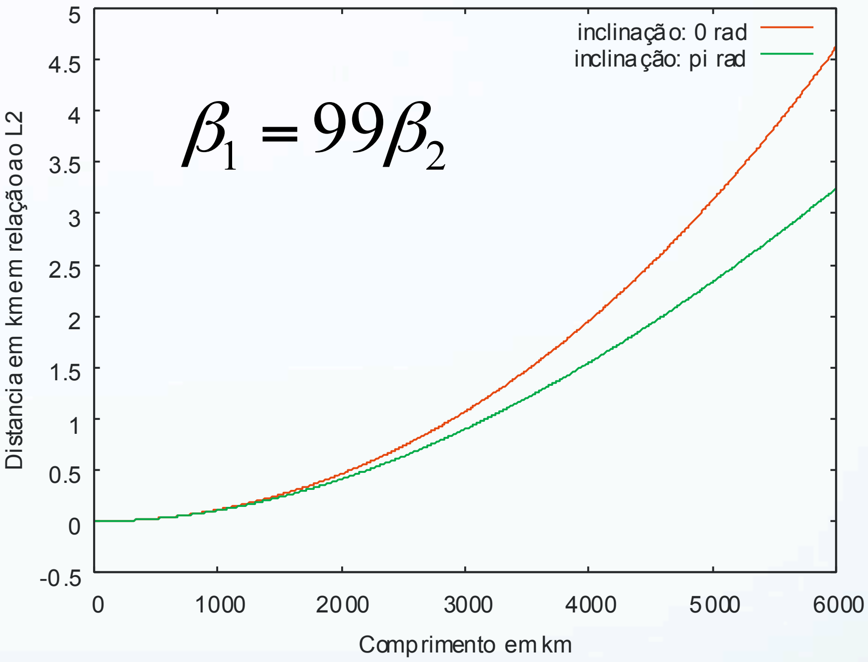


$$m_1 = m_2 \;\Longrightarrow\; \theta_{eq} = 0 \; ou \;\; \theta_{eq} = \pi \;\; ou \;\; \theta_{eq} = \frac{\pi}{2}$$



Equilibrium Points: Earth-Moon





Behavior for different tether length: Earth-Moon

initial conditions

$$x_0 = 0$$

$$x'_0 = 0$$

$$y_0 = 0$$

$$y'_0 = 0$$

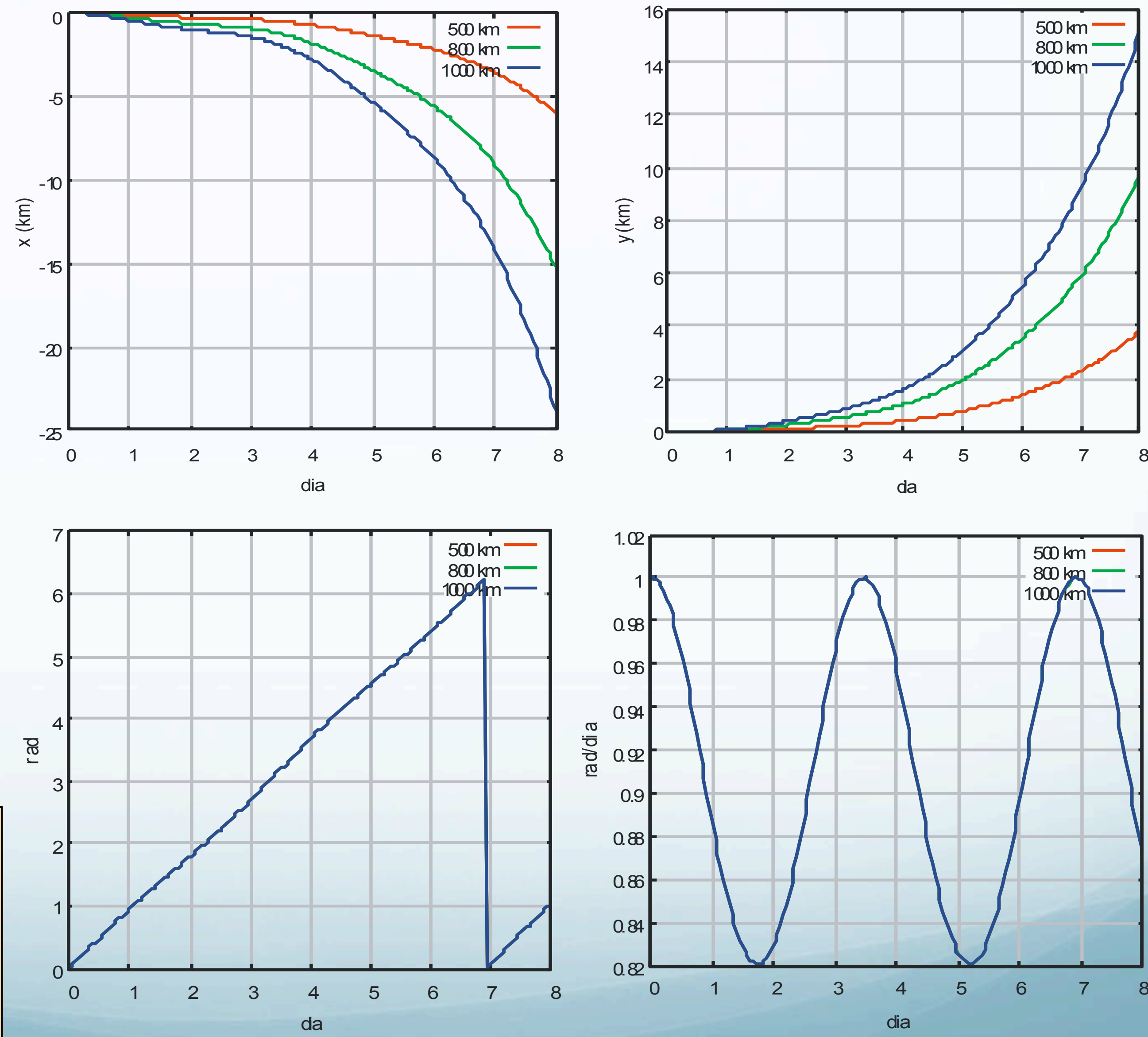
$$\theta_0 = 0, \pi/2$$

$$\theta'_0 = 10^{-3} / day$$

$$t = 8 days$$

$$l = 500/800/1000 km$$

$$m_1 = m_2$$



Lyapunov periodic orbits

→ **Periodic solutions in the vicinity of equilibrium points?**

- Linear stability
- Characterisc curve the of Lyapunov periodic orbits

Linear stability - a system with 6 EDO(s)

$$
\begin{cases}
\dfrac{dx}{dt} = \dot{x} \\
\dfrac{d\dot{x}}{dt} = +2\dot{y} - \left(\dfrac{1-\mu}{\hat{R}_1^3} \Lambda_1^{l,\theta} + \dfrac{\mu}{\hat{R}_2^3} \Lambda_2^{l,\theta} - 1 \right) (\hat{X}_0 + x) - \mu(1-\mu) \left(\dfrac{\Lambda_1^{l,\theta}}{\hat{R}_1^3} - \dfrac{\Lambda_2^{l,\theta}}{\hat{R}_2^3} \right) \\
\dfrac{dy}{dt} = \dot{y} \\
\dfrac{d\dot{y}}{dt} = -2\dot{x} - \left(\dfrac{1-\mu}{\hat{R}_1^3} \Lambda_1^{l,\theta} + \dfrac{\mu}{\hat{R}_2^3} \Lambda_2^{l,\theta} - 1 \right) y \\
\dfrac{d\theta}{dt} = \dot{\theta} \\
\dfrac{d\dot{\theta}}{dt} = \underbrace{-2\left(\dfrac{\hat{l}'}{\hat{l}} \right)(\theta' + 1)}_{=0} - \dfrac{1-\mu}{\hat{R}_1^3} \Omega_1^{l,\theta} - \dfrac{\mu}{R_2^3} \Omega_2^{l,\theta}
\end{cases}
$$

linear stability - Jacobian matrix

$$D_{\vec{x}}\vec{F} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{\partial F_1}{\partial x} & 0 & \frac{\partial F_1}{\partial y} & 2 & \frac{\partial F_1}{\partial \theta} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{\partial F_3}{\partial x} & -2 & \frac{\partial F_3}{\partial y} & 0 & \frac{\partial F_3}{\partial \theta} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{\partial F_5}{\partial x} & 0 & \frac{\partial F_5}{\partial y} & 0 & \frac{\partial F_5}{\partial \theta} & 0 \end{pmatrix}$$

Given the primaries and satellite masses

Jacobian matrix is a function of cable length only.

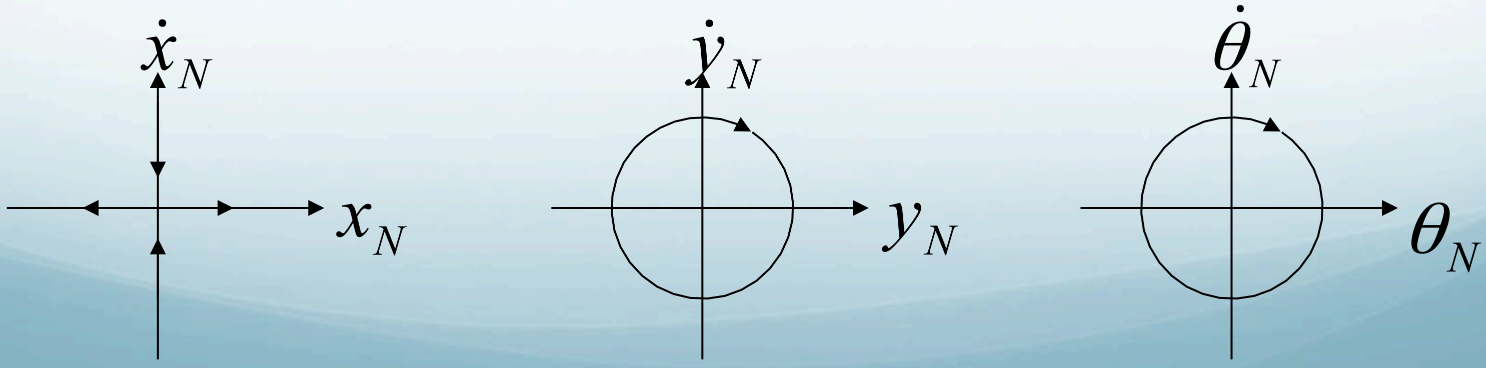
Linear stability - a system with 6 EDO(s)

$$
\begin{cases}
\dfrac{dx}{dt} = \dot{x} \\[2ex]
\dfrac{d\dot{x}}{dt} = +2\dot{y} - \left(\dfrac{1-\mu}{\hat{R}_1^3} \Lambda_1^{l,\theta} + \dfrac{\mu}{\hat{R}_2^3} \Lambda_2^{l,\theta} - 1 \right) (\hat{X}_0 + x) - \mu(1-\mu) \left(\dfrac{\Lambda_1^{l,\theta}}{\hat{R}_1^3} - \dfrac{\Lambda_2^{l,\theta}}{\hat{R}_2^3} \right) \\[2ex]
\dfrac{dy}{dt} = \dot{y} \\[2ex]
\dfrac{d\dot{y}}{dt} = -2\dot{x} - \left(\dfrac{1-\mu}{\hat{R}_1^3} \Lambda_1^{l,\theta} + \dfrac{\mu}{\hat{R}_2^3} \Lambda_2^{l,\theta} - 1 \right) y \\[2ex]
\dfrac{d\theta}{dt} = \dot{\theta} \\[2ex]
\dfrac{d\dot{\theta}}{dt} = \underbrace{-2\left(\dfrac{\hat{l}'}{\hat{l}} \right)}_{=0} (\theta' + 1) - \dfrac{1-\mu}{\hat{R}_1^3} \Omega_1^{l,\theta} - \dfrac{\mu}{R_2^3} \Omega_2^{l,\theta}
\end{cases}
$$

Earth-Moon: $\theta = 0$

$P(x_{eq},0,0,0,0,0)$ $l = 1000km$ $m_1 = m_2$

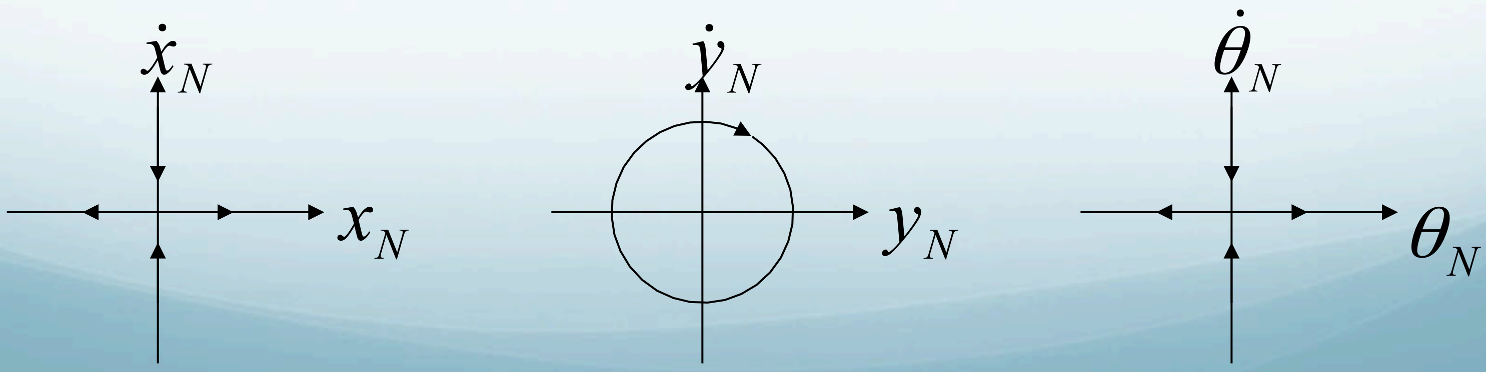
$$\vec{P}^{-1}(D_{\vec{x}}\vec{F})\vec{P} = \begin{pmatrix} 2.159 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2.159 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.862 & 0 & 0 \\ 0 & 0 & -1.862 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.093 \\ 0 & 0 & 0 & 0 & -3.093 & 0 \end{pmatrix}$$



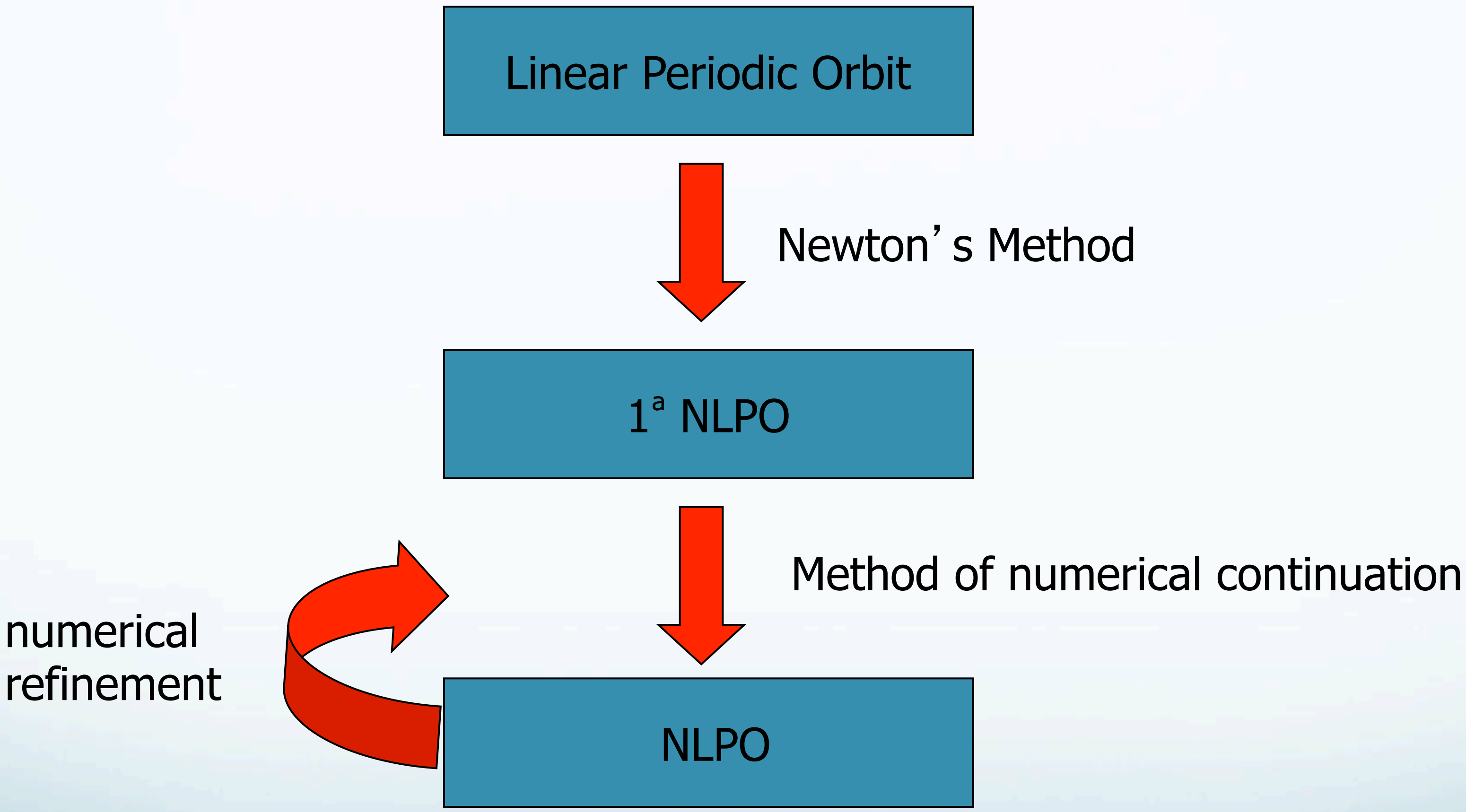
Earth-Moon $\theta = \frac{\pi}{2}$

$P(x_{eq},0,0,0,\frac{\pi}{2},0)$ $l = 1000km$ $m_1 = m_2$

$$\overleftrightarrow{P}^{-1}(D_{\vec{x}}\vec{F})\overleftrightarrow{P} = \begin{pmatrix} 2.159 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2.159 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.862 & 0 & 0 \\ 0 & 0 & -1.862 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3.093 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.093 \end{pmatrix}$$



Generation of Lyapunov periodic orbits



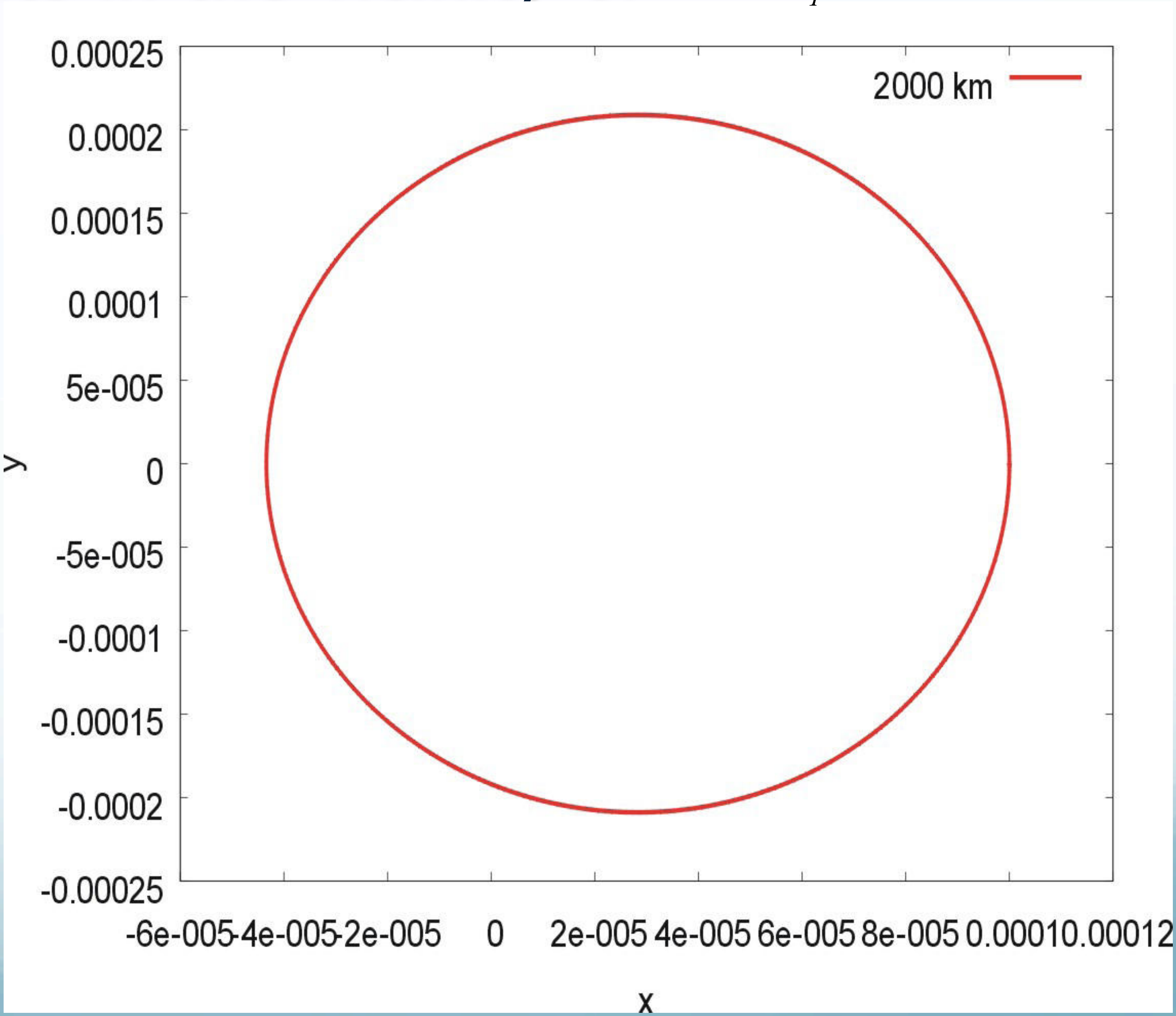
Numerical applications (Earth-Moon)

Periodic orbits in the vicinity of $P_0(x_{eq}, 0, 0)$

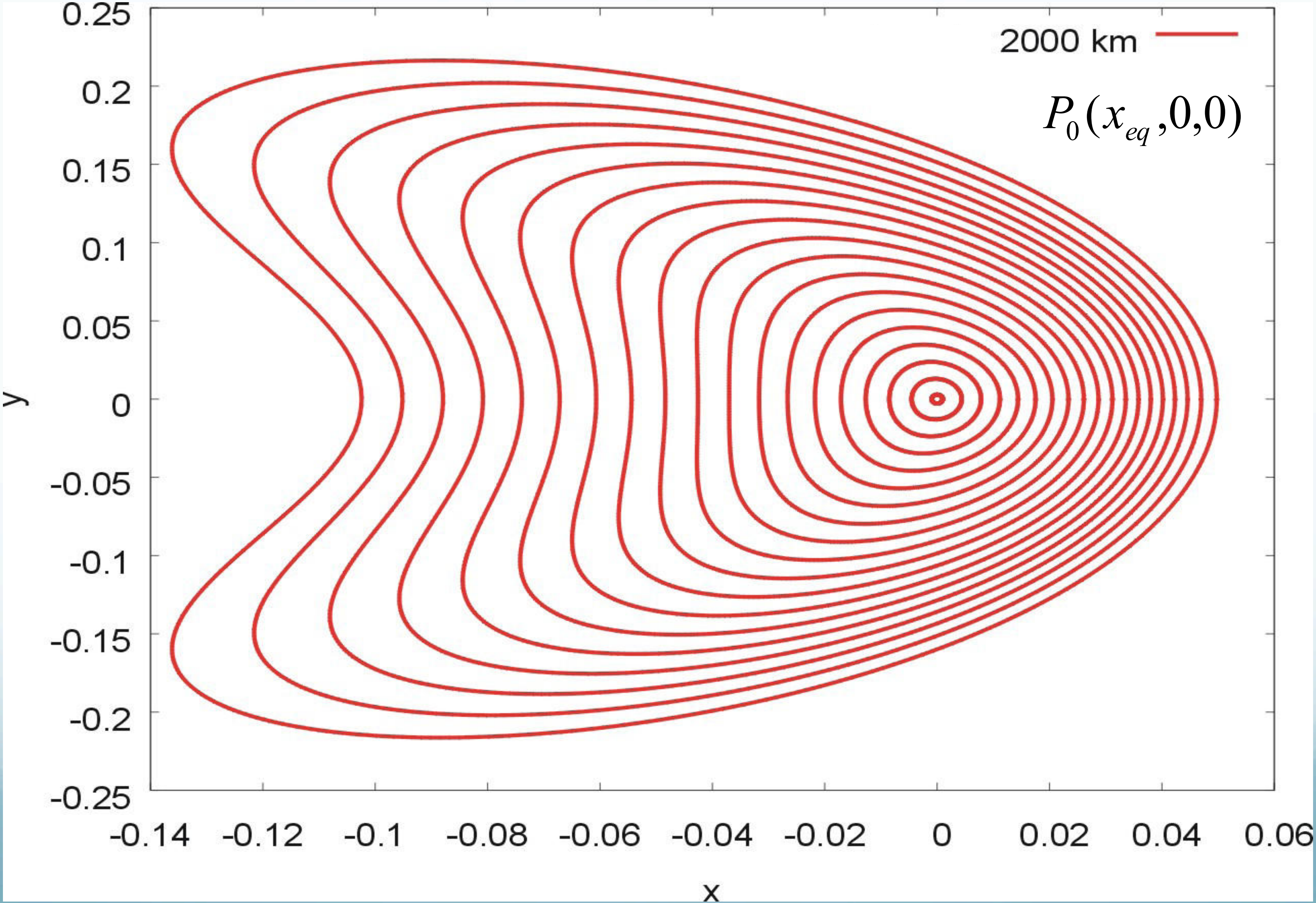
$$x_0 = 10.0015964019 \cdot 10^{-5}$$
$$\dot{x}_0 = 0.0000000000 \cdot 10^{-5}$$
$$y_0 = 0.0000000000 \cdot 10^{-5}$$
$$\dot{y}_0 = -38.897974167 \cdot 10^{-5}$$
$$\theta_0 = 0.0000000000 \cdot 10^{-5}$$
$$\dot{\theta}_0 = 0.0000000000 \cdot 10^{-5}$$

$$T = 3.3735104335$$

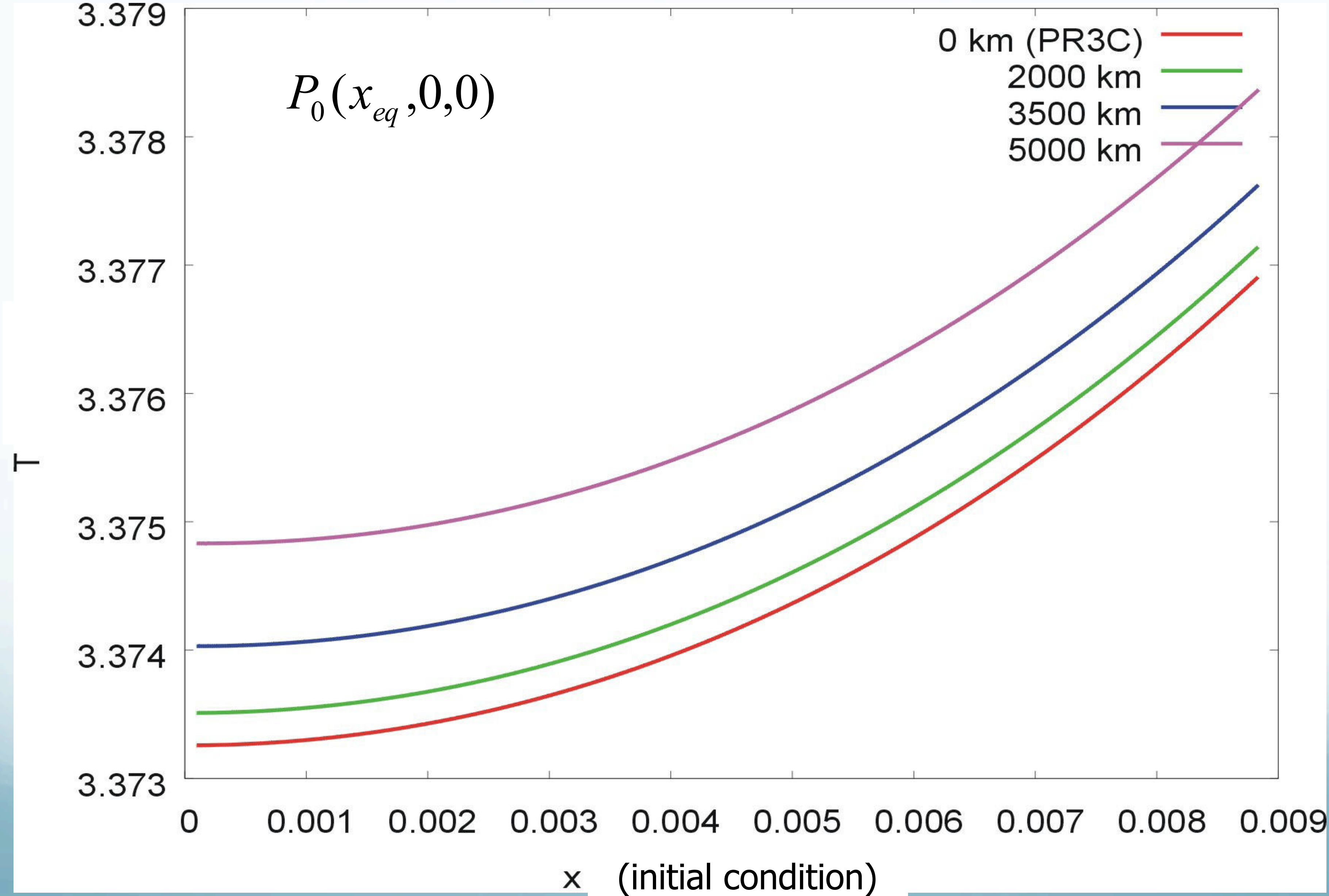
$$error \approx 10^{-12}$$



Some orbits of the Lyapunov family



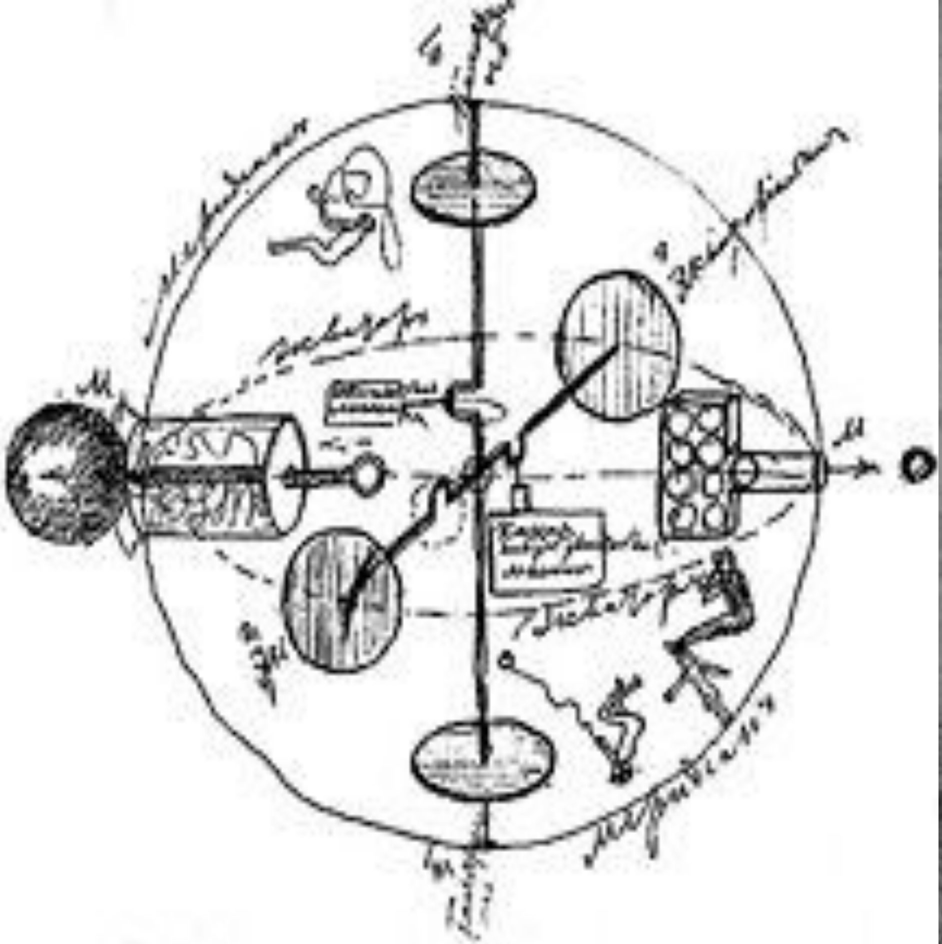
Comparison of the Characteristic Curves



Conclusions

- Our fourth order model is consistent with the RTBP
- The dynamics shows almost no variation with the masses
- The tether length is the most significative parammeter of the dynamics
- The Lyapunov orbits are not translations of RTBP
- The tether describes a Lyapunov orbit with theta=0

The linear coupling space-angle are tori, so a future work is to continue these tori.



Obrigada

L1 on the Earth side of the Moon is 56,000 km up from the surface, and L2 on the far side is 67,000 km up. In these positions, the forces of gravity and centrifugal force are equal, and as long as the system remained balanced (L1 and L2 are in unstable equilibrium along the line between Earth and Moon), it would remain stationary.Both of these positions are substantially farther up than the 36,000 km from Earth to geostationary orbit. Furthermore, the weight of the limb of the cable system extending down to the Moon would have to be balanced by the cable extending further up, and the Moon's slow rotation means the upper limb would have to be much longer than for an Earth-based system. To suspend a kilogram of cable or payload just above the surface of the Moon would require 1,000 kg of counterweight, 26,000 km beyond L1. (A smaller counterweight on a longer cable, e.g., 100 kg at a distance of 230,000 km — more than halfway to Earth — would have the same balancing effect.) Without the Earth's gravity to attract it, an L2 cable's lowest kilogram would require 1,000 kg of counterweight at a distance of 120,000 km from the Moon.The anchor point of a space elevator is normally considered to be at the equator. However, there are several possible cases to be made for locating a lunar base at one of the Moon's poles; a base on a peak of eternal light could take advantage of continuous solar power, for example, or small quantities of water and other volatiles may be trapped in permanently shaded crater bottoms. A space elevator could be anchored near a lunar pole, though not directly at it. A tramway could be used to bring the cable the rest of the way to the pole, with the Moon's low gravity allowing much taller support towers and wider spans between them than would be possible on Earth.

- The original space elevator, as Clarke acknowledges, was first described by Russian engineer Yuri Artsutanov in 1960, in an article in Pravda called "To the Cosmos By Electric Train." Since then, it's apparently been independently "reinvented" at least three times:

- (1) by a team from Scripps Institute of Oceanography and Woods Hole Oceanographic Institute (1966);

- (2) in 1969 by A.R. Collar and J.W. Flower in the Journal of the British Interplanetary Society;

- (3) and by Jerome Pearson of the Air Force Research Laboratory at Wright-Patterson Air Force Base (1975). It was hinted at, though not fully developed (for lack of a large enough envelope for calculations, he claims) by Clarke himself in 1963 in an essay for UNESCO on communications satellites.

http://www.niac.usra.edu/files/studies/final_report/472Edwards.pdf

http://gassend.net/spaceelevator/breaks/