Literature

A detailed discussion of the relation between symmetry and degeneracy, via a discussion of group theory can be found in:

J. P. Elliott and P. G. Dawber: *Symmetry in Physics*, Volume 1 and 2 (Cambridge University Press, Cambridge 1979)

and many other books devoted to group theory in physics. They also have a brief discussion of the Kepler (Coulomb) problem.
A more complete discussion of the Kepler problem is given in:

M. J. Englefield: *Group Theory and the Coulomb Problem* (Wiley, New York 1972)

Quick discussions of the symmetry aspects are given in some quantum theory texts.
See e.g., L. D. Landau and E. M. Lifshitz: *Quantum Mechanics: Non-Relativistic Theory*, revised third edition (Pergamon Press, Oxford 1977)

Delayed-Choice Experiments

A.J. Leggett

The phenomenon of ► "wave-particle duality" is at the heart of quantum mechanics, indeed has been described as "the one real mystery" of the subject. If we consider the standard Young's slits setup shown in Fig. 1, we may imagine for definiteness that the experiment is done with electrons (► Double-slit Experiment), then in the absence of "inspection" the probability of arrival of an electron on the final screen

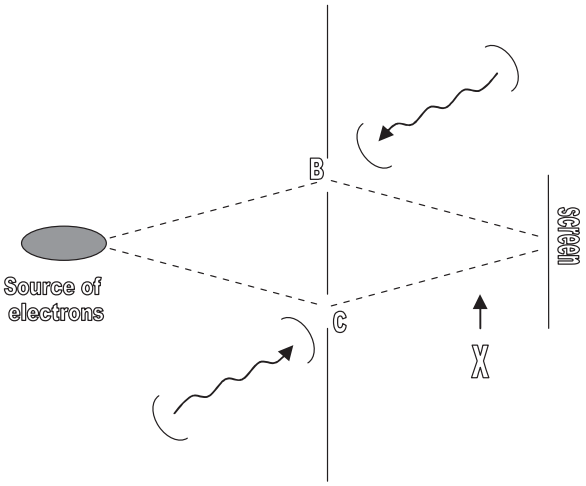

Fig. 1 The standard Young's slit setup. We may or may not choose to 'inspect' whether a given electron passes through slit B or slit C; the brackets indicate the optionality of the observation

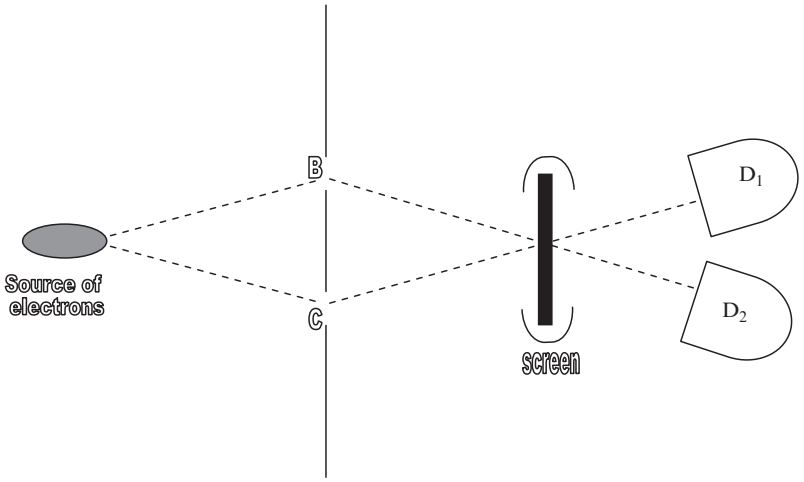

Fig. 2 An experiment illustrating "wave-particle duality" for photons. The brackets around the screen indicate that it may be either left in place (to indicate the "wave" aspect) or removed (to indicate the "particle" aspect)

shows the usual interference pattern – the electron appears to behave as a wave. If on the other hand we arrange to inspect which path is followed (e.g. by shining light on the intermediate slits as in the Heisenberg "gamma ray microscope" thought experiment ► Heisenberg microscope; which-way experiments), then the electron is always found, like a classical particle, to take one route or the other, and under these conditions no interference occurs at the final screen. If we replace the ► electrons with photons (► light quantum), we expect a similar duality to manifest itself; however, in this case, since it is very difficult to detect a photon without destroying it, it is more convenient to try to display the "particle" aspect by removing the final screen and replacing it by a pair of detectors as indicated in Fig. 2; again we will find that one detector or the other clicks, never both.

If D_1 clicks we can infer that the photon in question came through slit C, if D_2 clicks that it came through B. As is well known, Bohr interpreted experiments of this type to indicate that the very nature ("wave" or "particle") of elementary objects such as electrons or photons depends on the arrangement of the macroscopic experimental apparatus used to examine them; the arrangements needed to see wavelike behavior on the one hand and particle-like behavior on the other are always mutually exclusive ("complementarity"). This is particularly obvious in the example of the photon, and for definiteness I will from now on restrict myself to this case, although an entirely parallel discussion could be given for the case of an electron.

(See Consistent histories, Ignorance Interpretation, Ithaca Interpretation, Many Worlds Interpretation, Modal Interpretation, Orthodox Interpretation, Transactional Interpretation).

Is it necessary that the photon should as it were know in advance of entering the apparatus whether the latter has been set up in the "wave" configuration (Fig. 2) with the screen S in place or the "particle" one (S removed)? This question was already

raised by implication [1] within a few years of the birth of quantum mechanics, and in 1978 John Archibald Wheeler (1911–2008) [2] pointed out that it can be answered, at least in principle, by an experiment in which we leave the decision as to which configuration to use until after the ► wave packet representing the photon is well within the apparatus (let us say to the right of point X in Fig. 1). Such an experiment is called a "delayed-choice" experiment, and several have been done over the last 30 years, not only on photons but also on hydrogen atoms ► Bohr's atomic model and neutrons; without exception they have indicated that it does not matter whether the choice of configuration is made well in advance or only at the "last moment", the counting statistics are quite independent of this.

In the case of photons, if the dimensions of the apparatus are of the order of 3 m (a fairly typical value), the transit time is about 10 ns, and it is therefore essential, in conducting a meaningful delayed-choice experiment, that the time needed to make the "choice" should be substantially smaller than this. (For atoms and neutrons the requirement is somewhat less stringent). This obviously rules out the possibility of physically inserting or removing a screen as in Fig. 2; however, it turns out that one can get around this difficulty by exploiting the polarization degree of freedom. (For a different technique which does not rely on this, see below). The basic idea is to correlate (or decline to correlate) the path taken by the photon with its polarization, a choice which can be realized over a few nanoseconds with the help of a device such as a Pockels cell (which can rotate the plane of polarization by 90°).

A possible schematic realization is shown in Fig. 3: The photons emitted by the source are polarized (for example) in the plane of the paper, and in the absence of the Pockels cell (or if it is in place but not activated) this polarization is maintained throughout the experiment for both beams, so that they interfere at BS_2 with a relative phase which is controlled by the phase shifter. Thus, under these conditions the output of the detector D_1 (for example) is a periodic function of the phase difference introduced by the shifter ("wave" behavior). If on the other hand the Pockels cell is activated, the polarization of a photon in the lower beam is rotated out of

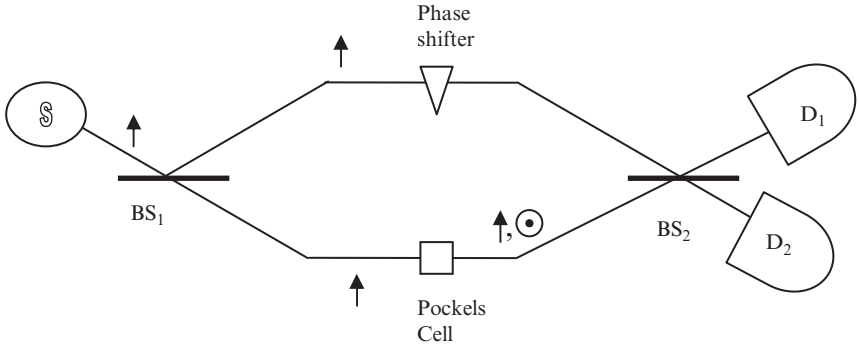

Fig. 3 Schematic realization of a polarization-mediated delayed-choice experiment. The notation to the right of the Pockels cell indicates that the polarization may, depending on our choice, be either in-plane or out-of-plane

the plane of the paper, so is perpendicular to that of the upper beam; the path taken by a given photon is now effectively "labelled" by its polarization. Under these conditions there can be no interference at BS_2 (which we assume is polarization-insensitive), and the output of detector D_1 is exactly the sum of what it would be for each of the two beams separately; since for each beam alone the output is independent of the position of the phase shifter, the total output of D_1 when the Pockels cell is activated is similarly insensitive to the latter ("particle" behavior). The crucial point is that the cell can be activated *after* the incoming photon wave packet has split at BS_1.

Over the last twenty years a number of experiments along these general lines have been done; the one closest to Wheeler's original proposal is probably that of ref. [3], which uses a setup similar though not identical[1] to that of Fig. 3. In this experiment the length of the interferometer was 48 m, and the choice as to whether or not to activate the switching cell was made by a quantum random number generator (QRNG) close to the far end; with this geometry the photon enters the future light cone of the random choice event long after it has passed the initial beam splitter. The use of the QRNG is designed to ensure that the photon has no way of "knowing" the choice ahead of time. The results are clear-cut: If one selects those photons for which the "wave" configuration was realized and plots the dependence of the output of one of the detectors on the phase shift between the two beams, one finds a well-defined sinusoidal pattern with visibility of 94%. If on the other hand one selects those photons which experienced the "particle" configuration, the corresponding plot is flat within experimental error.

An interesting variant of the "delayed-choice" experiment was reported in ref. [4]. The schematic setup is shown in Fig. 4: the "source" is prepared in such a way that there are nonzero mutually coherent amplitudes for a *pair* of photons to be emitted back-to-back by either of two regions A and B. Photon no.1 is registered by the screen S long before photon no.2 hits BS_1 or BS_2. The point of this arrangement is that any photon detected by D_3 (D_4) could only have come from source A(B); on the other hand, a photon arriving in D_1 or D_2 could have come from either source. Under these conditions, if we select only those photons 1 whose partners 2 were detected in (say) D_4 (let's call this the "D_4-correlated subensemble" of photons 1), we find that the distribution on the screen S is flat; on the other hand, if we select only those whose partners were detected in (say) D_1 ("D_1-correlated" subensemble), we obtain a well-defined fringe pattern (with a complementary pattern for those whose partners were detected in D_2). At first sight this is puzzling, since the detection of photon 1 on screen S took place well before the corresponding photon 2 "knew" whether it would be transmitted or reflected by BS1/2 and thus whether it will be detected by D_3/D_4 or by D_1/D_2.

In fact, there is no real paradox here (or in any of the other delayed-choice experiments); a consistent application of the quantum measurement axioms predicts

[1] Note in particular that in contrast to the setup of Fig. 3, in ref. [4] the activation of the electro-optical cell corresponds to the "wave" configuration and its non-activation to the "particle" configuration.

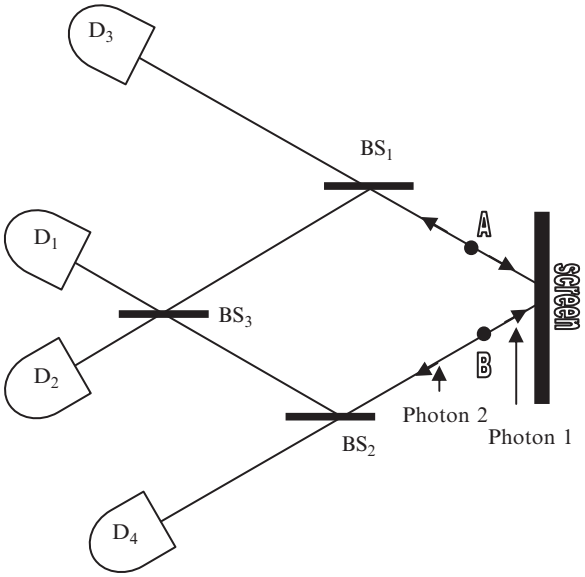

Fig. 4 The experimental arrangement of Kim et al. [4]

precisely the experimentally observed results. In particular, let us consider a case in which photon no.1 is detected at a point where the pattern corresponding to (say) the D_1-correlated subensemble has a node. When we say that the photon is "detected", we imply that it has induced a (quasi-) macroscopic event and thus satisfied what is usually considered the criterion for having undergone a "measurement". If at this point we apply the standard ► projection postulate to the two-photon system, we find that following the projection the ► wave function of photon 2 is automatically such that its amplitude to arrive in D_1 is zero, so everything is consistent. What the "delayed-choice" experiments really illustrate, in a spectacular way, is the pitfalls of applying the projection postulate at too early a stage in the game, while nothing has been registered at the macroscopic level and there is still a possibility of mutual interference of the possible alternatives.[2]

Primary Literature

1. K.F. von Weizsäcker: Ortsbestimmung eines Elektrons durch ein Mikroskop. Z. Phys. **70**, 114 (1931)
2. J.A. Wheeler: The "past" and the "delayed-choice" double-slit experiment, in A.R. Marlow, ed., *Mathematical Foundations of Quantum Theory* (Academic, New York 1978, pp. 9–48)

[2] This work was supported by the National Science Foundation through grant no.NSF-EIA-01-21568.

3. V.J. Jacques et al.: Experimental realization of Wheeler's delayed-choice gedanken experiment. Science **315**, 966 (2007)
4. Y.H. Kim et al.: Delayed "choice" quantum eraser. Phys. Rev. Lett. **84**, 1 (2000)

Secondary Literature

5. R.P. Feynman: *The Character of Physical Law* (MIT, Cambridge 1965)
6. N. Bohr: *Atomic Physics and Human Knowledge* (Wiley, New York 1958)

Density Matrix

Leslie Ballentine

A matrix representation of the ► *state operator*. So named because in the *position* basis its diagonal elements are equal to the position probability density. This name is older than the modern term *state operator*, and is still frequently used in its place, especially in many-electron theory and ► quantum chemistry. The name *density matrix* is not entirely accurate, since in the position basis it is not really a matrix, but rather a function of two continuous variables. If a discrete basis is chosen (such as the *spin* basis), then it becomes a genuine matrix, but its diagonal elements are *probabilities* rather than densities. ► States, pure and mixed, and their representation.

Density Operator

Werner Stulpe

Density operator, an operator used to describe (mixed) quantum states. A *density operator* [1–6], also called *statistical operator* or – somehow misleading – density matrix, is a positive trace-class ► operator ρ of trace 1 acting in some separable complex ► Hilbert space $\mathcal{H}$; i.e., ρ is a linear operator defined on $\mathcal{H}$ with values in $\mathcal{H}$ that satisfies $\rho = \rho^*$, $\langle\phi|\rho\phi\rangle \geqslant 0$ for all $\phi \in \mathcal{H}$, and $\operatorname{tr}\rho = \sum_i \langle\phi_i|\rho\phi_i\rangle = 1$, $\phi_1, \phi_2, \ldots$ being a complete orthonormal system in $\mathcal{H}$. In particular, ρ is a compact self-adjoint ► operator; in consequence, a density operator has the spectral decomposition $\rho = \sum_i \lambda_i P_{\chi_i}$ (► self-adjoint operator) where $\lambda_1, \lambda_2, \ldots$ are the nonzero eigenvalues of ρ, counted according to their multiplicity and arranged according to $\lambda_1 \geqslant \lambda_2 \geqslant \ldots > 0$, $\sum_i \lambda_i = 1$, $\chi_1, \chi_2, \ldots$ is an orthonormal system