maxima in the interference pattern, and the angular width $\delta\alpha$ of each maximum, are

$$\Delta\alpha \approx (\lambda/D)\ , \qquad \delta\alpha \approx (d/D)\Delta\alpha \ll \Delta\alpha\ , \tag{4}$$

where λ is the wavelength. As detector of the transmitted light a photographic emulsion will do. If the light intensity is very low, and the plate is exposed for a sufficiently short time, a few dark spots will be seen, each due to a photochemical reaction triggered by a single photon. As the length of exposure is increased, the number of spots will increase and they will acquire a density distribution that approaches that of the classically predicted interference pattern. In short, the wave picture gives the correct probability for where a photon is detected.

If we accept the proposition that photons have corpuscular properties, this would presumably imply that the "size" of a photon is microscopic and small compared to a single slit. Then it is only natural to ask what would be observed if the plane wave that covers the whole grating were replaced by sequential exposures that illuminate just one slit at a time. According to classical optics, however, this would produce a succession of diffraction patterns from individual slit, which have an angular width λ/d, i.e., cover many of the maxima in the diffraction pattern when the whole grating is illuminated.

This consideration, and others of a similar vein, lead to an important conclusion:

> *In any setup that allows light to traverse different paths, these paths can either be combined coherently to form an interference pattern in which case the experiment cannot reveal which path a photon follows, or the apparatus can be modified to determine which path is followed but this destroys the interference pattern.*

This is yet another illustration of complementarity: an arrangement designed to manifest one of the properties of a phenomenon expected from the classical viewpoint contains the possibility of observing at least some of the other classical properties. In this instance, the complementary properties are the wave property of interference and the particle property of path.

The elegant experiment sketched in Fig. 1.1 further elucidates the complementarity of the concepts of path and interference. Here two trapped and effectively stationary Hg^+ ions are illuminated by a laser beam. The photons that comprise this beam are scattered by the ions and then detected. Because the probability of scattering is small, the experiment is in effect a succession of individual photon collisions by one or the other of the two ions. If, for a moment, we assume the ions to be structureless, and that there is no such thing as polarization, the scattered light would have the same angular distribution as in the classical Young two hole interference experiment. The ions are, however, spin $\frac{1}{2}$ objects. Furthermore, the laser produces linearly polarized light, and the experiment can select scattered photons of various linear polarizations. As a consequence, by judicious choices of the polarizations of the incident and scattered photons, the events can be separated into two categories:

1. those in which there is *no possibility* that either ion underwent a spin flip ;
2. those in which one of the ions *must* have undergone a spin flip.

In case 1, it is impossible to determine which of the two ions was responsible for scattering, so the path taken by the photon is unknown in principle. In case 2, on the

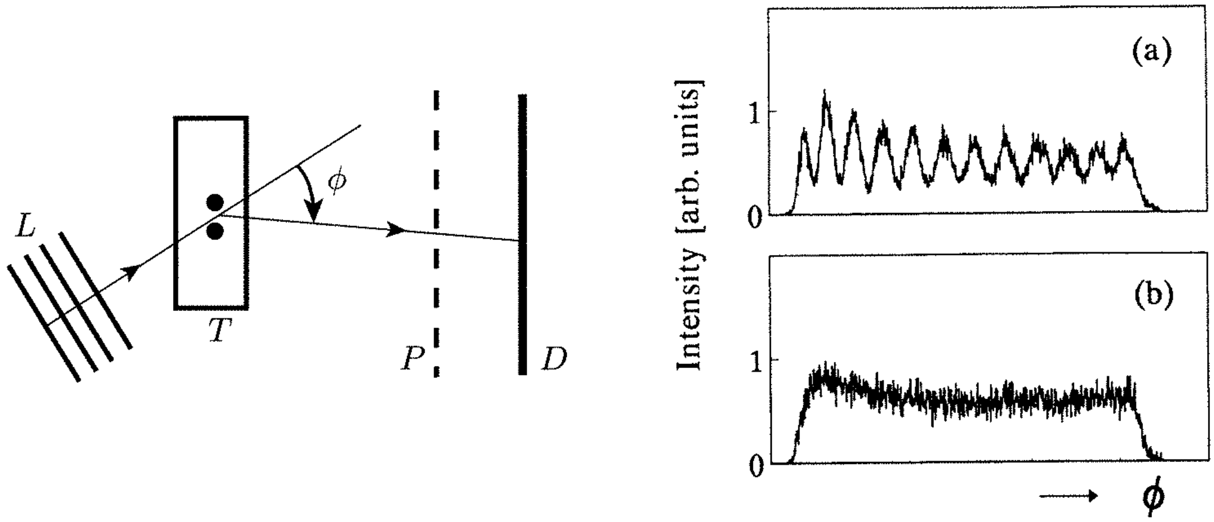

FIG. 1.1. Sketch of an interference experiment. L is a linearly polarized laser beam, T a trap that holds two Hg^+ ions effectively at rest, P a polarizer, and D a detections screen. (a) Events in which spin flip is excluded and the two possible photon paths are indistinguishable. (b) Events in which one of the ions must have had a spin flip which, in principle, determines the photon path. From U. Eichmann, J.C. Berquist, J.J. Bollinger, J.M. Gilligan, W.M. Itano, D.J. Wineland and M.G. Raizen, *Phys. Rev. Lett.* **70**, 2359 (1993). See also W.M. Itano et al., *Phys. Rev. A* **57**, 4176 (1998).

other hand, an examination of the spins of the ions after the photon has scattered would select which one caused the scattering and thus determine the photon's path. Hence the events in category 1 should show the Young interference pattern, while those in category 2 should not. The data confirming this is shown in Fig. 1.1.

Note well that in this experiment there is no instrument that actually measures the spin of the ions after scattering. This fact establishes two important points:

- The question of whether there is interference is settled solely by whether it is impossible or possible to determine which path is taken by a photon. Whether or not the determination is actually made does not matter.

- The "destruction" of the interference pattern arising from collisions where one ion has undergone a spin flip cannot be ascribed to some "irreducible disturbance caused by measurements" carried out on these photons. The collisions which do not cause a spin flip, and do produce the interference pattern, disturb the photons as much or as little as those in which there is a spin flip. To repeat the preceding point, all that counts is whether or not evidence exists that can reveal the photon's path.

For clarity's sake, the absence or presence of interference was just stated as a crystal clear distinction. The full story is that the visibility of the interference pattern diminishes as the confidence grows with which the photon path becomes knowable. For this purpose, consider diffraction by two holes in an opaque screen, with one of the holes being partially absorbing.[1] The amplitude and intensity are then

$$\varphi = ae^{ikL_1} + be^{ikL_2} , \quad |\varphi|^2 = a^2 + b^2 + 2ab\cos[(d/L)kx] , \tag{5}$$

[1] D.M. Greenberger and A. YaSin, *Phys. Lett. A* **128**, 391 (1988).

where $L_{1,2}$ are the distances from the two holes to the observation point, d is the separation of the holes, and L the separation between the screen to the parallel detection plane (assuming $d/L \ll 1$). If there is no absorber, a equals b. The visibility of the diffraction pattern can be defined as

$$V = \frac{|\varphi|^2_{\rm max} - |\varphi|^2_{\rm min}}{|\varphi|^2_{\rm max} + |\varphi|^2_{\rm min}} = \frac{2ab}{a^2+b^2} \ . \tag{6}$$

If two detectors are placed just behind the holes, they will register with rates proportional to a^2 and b^2. The probabilities that any one photon takes one or the other path are then a^2/N and b^2/N, with $N = a^2 + b^2$, and their difference is

$$\Delta = \frac{a^2-b^2}{a^2+b^2} \ . \tag{7}$$

Thus there is a smooth transition as $|a/b|$ departs from $a = b$, when the two paths are equally likely ($\Delta = 0$) and the visibility has its maximum value of $V = 1$, to zero visibility as $|b/a| \to 0$ when it is certain that every photon takes one of the two paths, i.e., $|\Delta| = 1$. Moreover,

$$V^2 + \Delta^2 = 1 \ , \tag{8}$$

so there is a relationship between the degrees to which the complementary wave-like and particle-like features can be in evidence simultaneously. As V is linear in b, a substantial diffraction pattern survives even when $b^2 \ll a^2$ and it is reasonably safe to bet on the path of the next photon.

A similar conclusion applies to the famous debate in which Einstein proposed to determine which of two slits in a plate each particle traverses in contributing to a diffraction pattern by measuring the recoil of the plate, which Bohr showed is incorrect because the uncertainty principle must also be applied to the plate (see the discussion following Eq. 17). A more elaborate analysis shows that when measurement of the plate's recoil provides incomplete knowledge of which slit is traversed there is also a visible diffraction pattern.[1]

(b) The Uncertainty Principle

The uncertainty principle emerges when the Einstein relations for the photon's momentum and energy are combined with the assumptions that the energy density of classical electrodynamics gives the probability of detecting individual photons in a given space-time volume. The classical electric field $\boldsymbol{E}(\boldsymbol{r},t)$ in empty space satisfies

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\boldsymbol{E} = 0 \ , \qquad \boldsymbol{\nabla}\cdot\boldsymbol{E} = 0 \ , \tag{9}$$

with boundary conditions appropriate to the apparatus in question; so does the magnetic field. The most general solution of (9) is

$$\boldsymbol{E}(\boldsymbol{r},t) = \int d\boldsymbol{k}\, e^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}\, \boldsymbol{a}(\boldsymbol{k}) \ , \tag{10}$$

[1]W.K. Wooters and W.H. Zurek, *Phys. Rev. D* **19**, 473 (1979); *WZ*.