On the use of a virtual Mach–Zehnder interferometer in the teaching of quantum mechanics

Alexsandro Pereira[1], **Fernanda Ostermann**[2] and **Cláudio Cavalcanti**[2]

[1] Graduation Programme in Physics Education, Federal University of Rio Grande do Sul, Porto Alegre 15051, Brazil
[2] Physics Institute, Physics Department, Federal University of Rio Grande do Sul, Porto Alegre 15051, Brazil

E-mail: alexsandro.pereira@ufrgs.br

Abstract
For many students, the conceptual learning of quantum mechanics can be rather painful owing to the counter-intuitive nature of quantum phenomena. In order to enhance students' understanding of the odd behaviour of photons and electrons, we introduce a computational simulation of the Mach–Zehnder interferometer, developed by our research group. An example of an instruction session based on the use of the virtual Mach–Zehnder interferometer is also presented.

Introduction

This work is focused on the virtual Mach–Zehnder interferometer, a didactical tool which was developed by our research group (Ostermann *et al* 2006). The Mach–Zehnder interferometer is an experimental arrangement, independently developed by Ludwig Mach and Ludwig Zehnder around 1892, which demonstrates light interference phenomena by means of division of a light beam. Even though it has seen considerable use in experimental physics research and technological applications (for example, see Dimitrova and Weis 2008, and Kanseri *et al* 2008), the Mach–Zehnder interferometer is rarely mentioned in the physics textbooks that are mostly used at undergraduate level. This makes the interferometer quite unfamiliar to high school physics teachers. Nevertheless, some authors have remarked on the didactical potential of the Mach–Zehnder interferometer in the context of quantum physics teaching (Adams 1998, Pessoa Jr 1997, Scarani and Suarez 1998). As asserted by Müller and Wiesner (2002), the interferometer helps the students to perceive, from the beginning, how quantum phenomena deviate from our classical everyday experience.

In the UK, wave–particle duality is an optional component of the AQA AS/A level specification (Turning Points in Physics). In quantum physics teaching, this subject is usually introduced through either the experiments on electron diffraction or the double-slit experiment, acclaimed by Richard Feynman in his book series published in the 1960s (Feynman *et al* 1963). These two experiments are arrangements rather familiar to physics teachers, although studies have shown that many students encounter serious difficulties in trying to understand the light interference phenomena demonstrated by Young's experiment (Planinšič and Sliško 2005). Moreover, results from other studies concerning the teaching of quantum physics have shown that many students tend to formulate a quite vague notion of wave–particle duality, viewing light as particles that move in waves (Olsen 2002). In the double-slit experiment for electrons, the interference pattern could be classically explained as being the result of the Coulombic interaction between the electron beam, on one hand, and the atoms located at the borders of the slits, on the other. In the Mach–Zehnder interferometer, the interference phenomenon is caused by a phase difference between the laser components introduced by the combination of mirrors and beam splitters (Zetie *et al* 2000), as seen in figure 1. Didactically, the Mach–Zehnder interferometer is more appropriate for the discussion of quantum interference phenomena, since it naturally leads the students to question which path the photon has taken (Scarani and Suarez 1998).

The Mach–Zehnder interferometer

In figure 1 a coherent monochromatic light beam, with intensity I_0, strikes the beam splitter S_1, which splits the incident beam into two components: one transmitted (labelled with T) and another reflected (labelled with R), travelling on paths I and II, respectively, both coherent, and with the same intensity $I_0/2$. After being reflected by mirrors M_1 and M_2, each of these components is again subdivided into two components by the second beam splitter S_2 (paths a and b), before hitting the detectors D_1 and D_2. The labels R and T show the number of reflections and transmissions experienced by these components after each interaction with the beam splitters and mirrors.

The classical interference phenomenon arises from the superposition of waves. In both detectors there is a superposition of the amplitudes of the two components of the laser beam, each one with an intensity $I_0/4$. Considering waves travelling in the positive x direction, the electric field of the laser beam emitted by the source can be

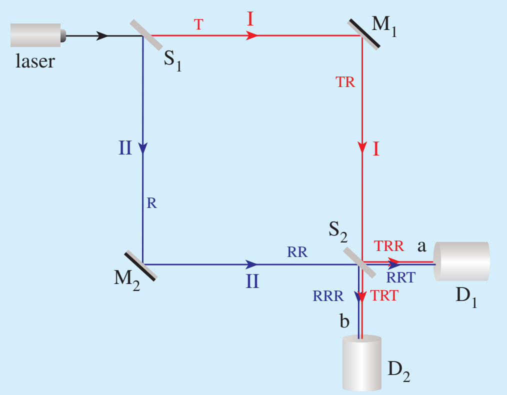

Figure 1. Scheme of the Mach–Zehnder interferometer.

mathematically expressed as

$$E = E_0 \hat{x} \cos(kx - \omega t) \tag{1}$$

where $\hat{x}$ is the unitary vector along the positive x direction, $k = 2\pi/\lambda$ is the wavenumber (λ is the wavelength) and ω is the angular frequency. The intensity of an electromagnetic wave is determined using the following expression:

$$I = \frac{1}{c\mu_0}\overline{E^2} = \frac{1}{c\mu_0}\overline{\left(E_0 \hat{x} \cos(kx - \omega t)\right)^2} \tag{2}$$

where the bar denotes the time average taken over a period $T = 2\pi/\omega$. It is possible, therefore, to establish the following relations to the laser beam:

$$\begin{gathered} I_0 \sim E_0^2 \\ \frac{I_0}{4} \sim \frac{E_0^2}{4} = \left(\frac{E_0}{2}\right)^2. \end{gathered} \tag{3}$$

For an incidence angle of 45°, the phase shift between the transmitted and the reflected light beam components is $\pi/2$, which corresponds to a path length difference $\lambda/4$, where λ is the wavelength (Degiorgio 1980). These components originate from the interaction between the wave and the beam splitters or mirrors. Thus, the beam components that hit D_1 experience two consecutive reflections and one transmission (labelled with TRR and RRT in figure 1; respectively paths I-a and II-a), remaining in phase. Therefore, we observe a constructive

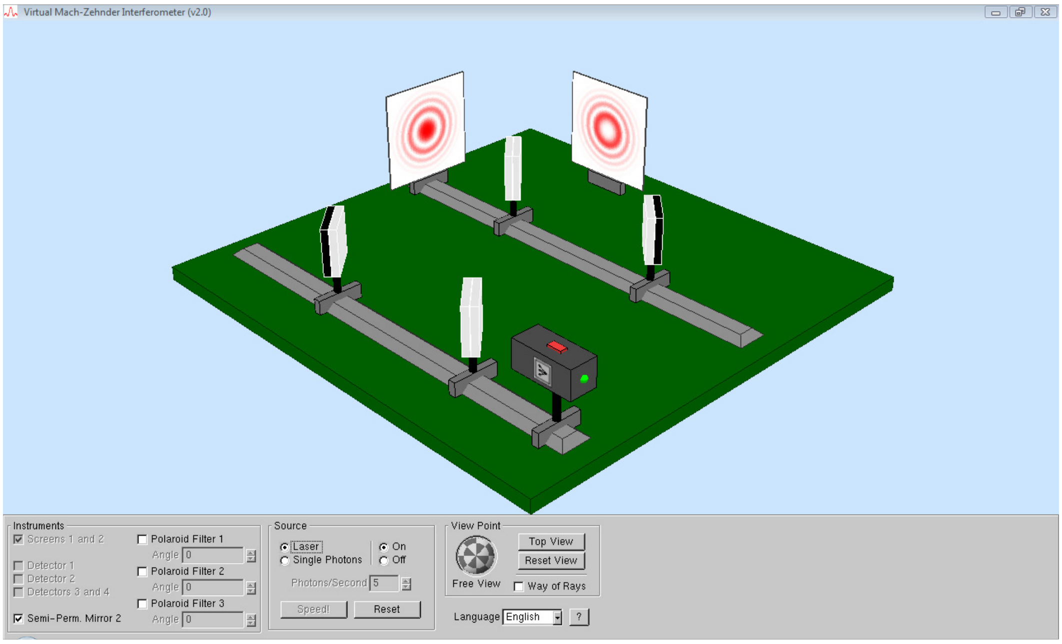

Figure 2. Ring pattern on the screens.

interference pattern in D_1, as we show below:

$$E = \frac{E_0}{2}\hat{x}\cos(kx - \omega t + \pi) + \frac{E_0}{2}\hat{x}\cos(kx - \omega t + \pi) = E = -E_0\hat{x}\cos(kx - \omega t). \quad (4)$$

That is, all the intensity of the laser beam in D_1 is recovered, but with a phase inversion.

Analysing the path taken by the two components of the light beam travelling in the D_2 direction, we see that one of the components experiences only one reflection (labelled with TRT in figure 1; path I-b), while the other experiences three consecutive reflections (labelled with RRR in figure 1; path II-b). Then, we can conclude that the beam components that hit D_2 are delayed by π, which corresponds to a destructive interference pattern. Thus, we have all of the beam energy detected in D_1 and none in D_2, as shown below:

$$E = \frac{E_0}{2}\hat{x}\cos\left(kx - \omega t + \frac{\pi}{2}\right) + \frac{E_0}{2}\hat{x}\cos\left(kx - \omega t + \frac{3\pi}{2}\right) = 0. \quad (5)$$

The computational simulation: the virtual Mach–Zehnder interferometer

Figure 2 shows the layout of the virtual Mach–Zehnder interferometer operating in classical mode. The laser source emits a light beam with a small angular aperture, which leads to a ring pattern formation on both screens. Notice that the laser beam does not hit the central point of screen 2 (as predicted in the previous section). We assume the laser beam as linearly polarized in the horizontal direction.

Figure 3 shows the destruction (on screen 2) and restitution (on screen 1) of the interference pattern due to the presence of the polaroid filters 1, 2 and 3. The polarization directions of the polaroid filters, in this case, are respectively oriented at 45°, 135° and 90°, according to the horizontal direction.

The virtual Mach–Zehnder interferometer has some check-boxes to control the simulation. In the *Instruments* group (figure 4), it is possible to remove and replace the second beam splitter (bottom left corner) by deselecting the *Semi-Perm. Mirror 2* check-box. In the same group it is possible to operate with the polaroid filters 1, 2 and 3, by selecting the corresponding check-boxes and

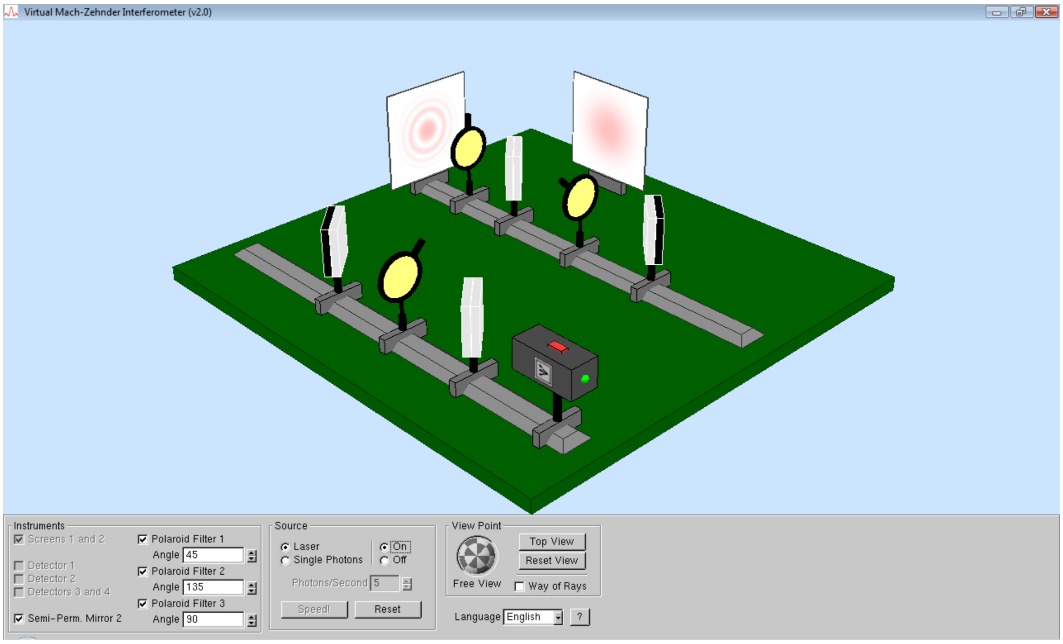

Figure 3. Reconstitution of the ring pattern on screen 1.

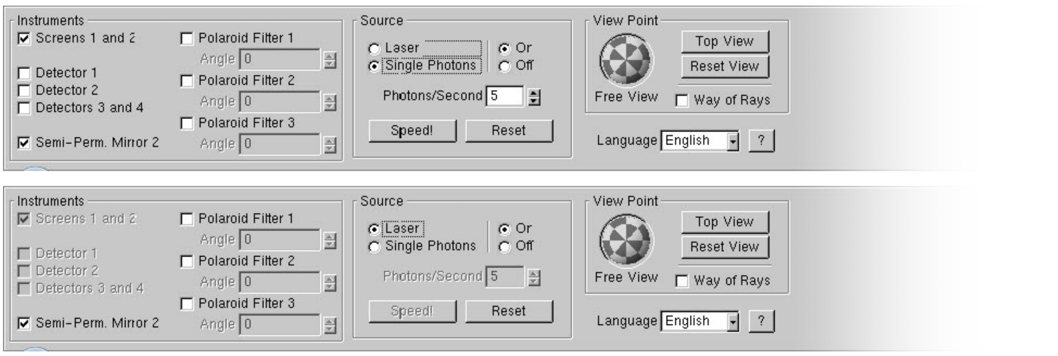

Figure 4. Control panel: quantum mode (upper) and classical mode (lower).

inserting the orientation (in degrees) in the field below each check-box.

In the *Source* group, we can switch the source on and off, and also have the *Laser* and *Single Photons* check-box options. For single photons the virtual Mach–Zehnder interferometer makes available, in the *Instruments* group, four photon detectors that can replace the screens and the polaroid filters 1 and 2, as displayed in figure 5. It is also possible to control the photon emission rates (option *photons/second*), accelerating the counting of the photons collected at the screens and detectors.

In the *View Point* group, it is possible to select the perspective for seeing the simulation. Below the *View Point* group is the option to choose the preferred program language (English, Spanish or Portuguese).

Instruction based on the use of the virtual Mach–Zehnder interferometer

As an example of instruction based on the use of our computational simulation, we present a short case study involving fourteen pre-service physics teachers. In our study, we took into account

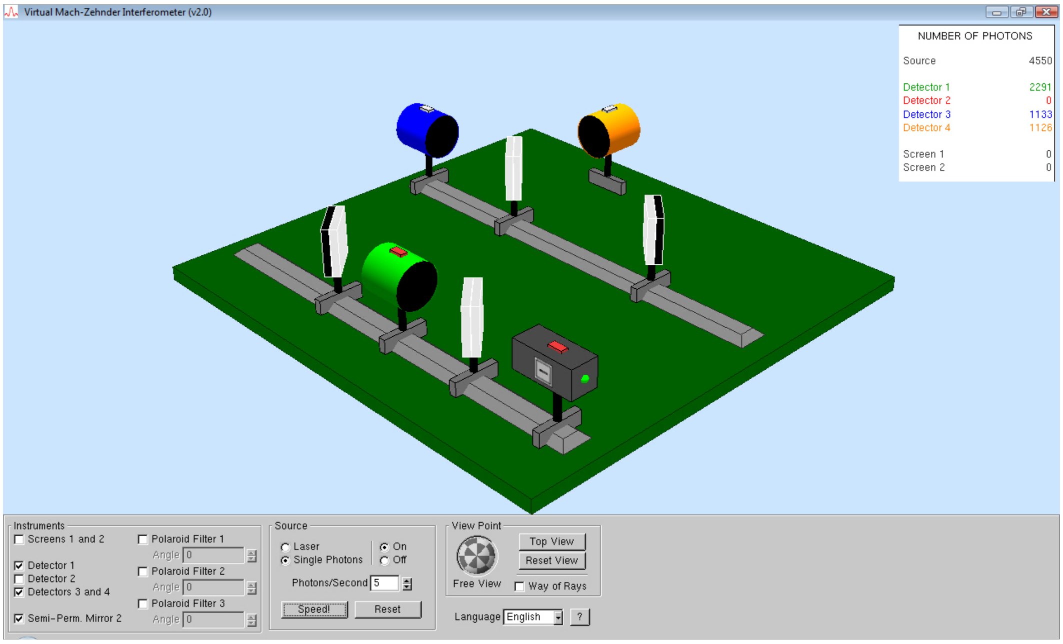

Figure 5. Detectors replacing polaroid filters and screens.

the assumption that most students conceive of the quantum objects as essentially being 'classical particles'(Budde *et al* 2002). The data gathered in this study are from a wider investigation involving the teaching of quantum physics in the social–cultural perspective (Pereira 2008).

Teaching approach based on analogies

The use of analogies is very common in science teaching and has provided considerable help to physics teachers. Some authors, however, remark on the fact that this resource is more useful when used explicitly (Taber 2001). In order to make the odd behaviour of photons a little more intuitive, we established an analogy between quantum physics and wave optics. By the analogy with wave optics we mean to give a stronger emphasis to the wave aspects of quantum phenomena, instead of considering the photons as light corpuscles (Marburger 1996). In some senses, this approach is in accordance with the 'Berling Concept of Quantum Physics', especially in the case of electron diffraction (Fischler and Lichtfeldt 1992).

In other words, by establishing an analogy between quantum physics and wave optics we mean to use the correspondence principle to evaluate the quantum phenomena. According to this principle, we can infer the following rule for single photons.

'*The experiments with single photons (quantum mode) gradually reproduce the same results as ones with a laser beam (classical mode).*'

In our approach, the quantum phenomenon is the physics of waves with low intensity, when the corpuscle properties of light start to appear (Pessoa Jr 2005). The intensity of the laser beam, in the classical mode, corresponds to the probability of finding a photon in some particular region in the quantum mode.

Initial hypothesis on the use of the virtual Mach–Zehnder interferometer

According to the results of other studies (Budde *et al* 2002, Olsen 2002), it is possible to develop teaching hypotheses on the use of the virtual Mach–Zehnder interferometer in quantum physics lessons. These hypotheses allow us to manage some activities that can avoid inadequate assimilation by students of some quantum physics concepts.

Teaching hypothesis 1: an analogy between quantum physics and wave optics. The virtual

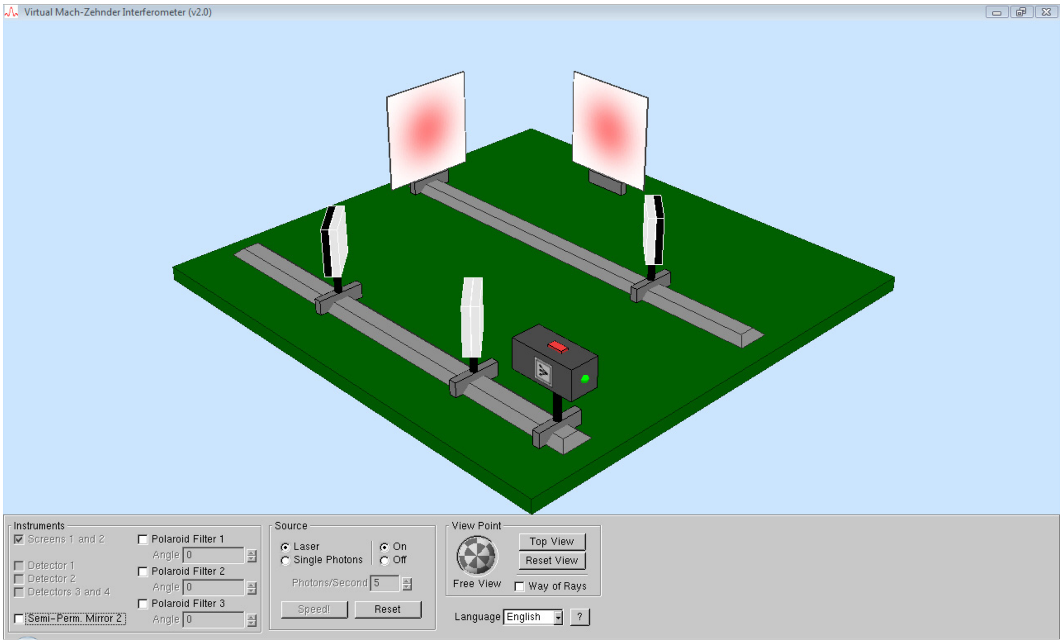

Figure 6. Laser beam division.

Mach–Zehnder interferometer functioning in both classical and quantum modes can help students to use the correspondence principle, once it shows that experiments using single photons reproduce gradually the same results as experiments using a laser beam.

Teaching hypothesis 2: the conceptual problem concerning the path of the photon. The experimental arrangement of the Mach–Zehnder interferometer can help students to glimpse the conceptual problem of the photon's path choice, which can highlight the notion that quantum objects and classical particles have quite different behaviours.

Didactical strategy

In September 2007, we implemented a didactical activity focusing on the exploration of the virtual Mach–Zehnder interferometer for a small group of fourteen students from the Physics Education course at the Federal University of Rio Grande do Sul, Brazil. The aim of this instruction as a whole was to help the students to predict qualitatively the odd behaviour of the photon. The present students (eleven in total) were separated into five groups (four pairs and one trio). A short guide was written to direct the students throughout the task. They received five computers with the computational simulation installed, microphones and sound recorders. The dialogues established within each group were recorded and their transcriptions were later analysed.

The analysis of discourse used in this study is based on the socio-cultural framework (Bakhtin 1997, Bakhtin 2006, Lemke 1997, Vygotsky 1994, 1998, Wertsch 1993) and its outcomes will not be discussed in this work. Instead, we present an example of dialogue established by two students, as they progress in the task. It was possible to identify the successful use of the analogy between quantum physics and wave optics.

Dialogues between Alice and Bob

In the following we present a synthesis of the didactical task performed by the students Alice and Bob (fictitious names). Many of the transcribed dialogues corroborate the hypothesis summarized in the previous section.

Virtual interferometer operating in classical mode

Initially, the students removed the second beam splitter and turned on the laser source, as shown in figure 6.

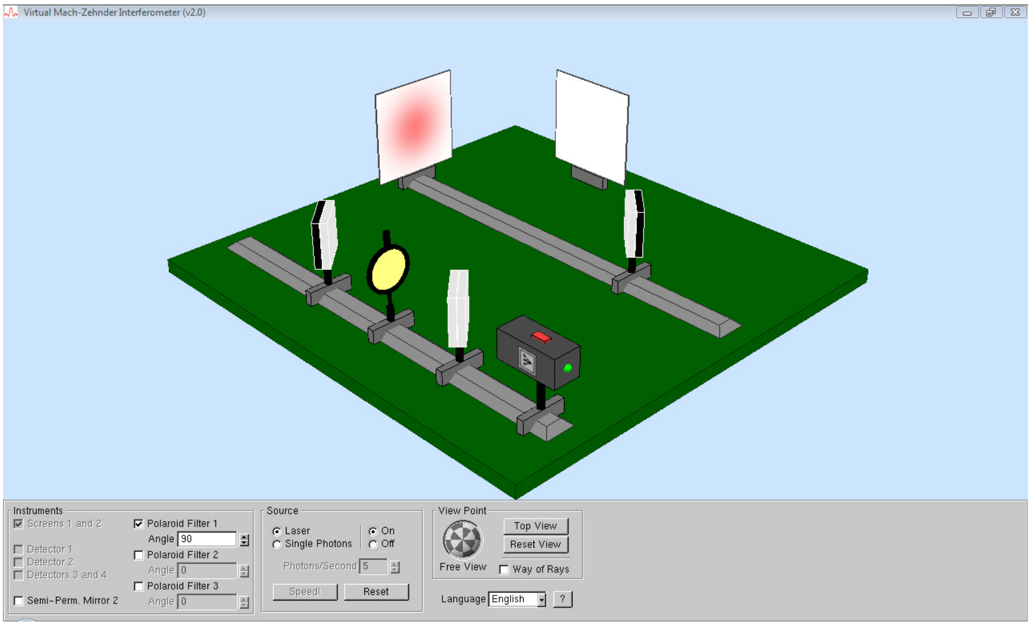

Figure 7. Determining the laser beam polarization.

They could then ascertain that the second beam splitter is the element responsible for the interference pattern formed by the two laser components, as shown in the following dialogue.

Dialogue 1

(1) Alice: There is no interference. These two light beams will cross each other and will not interfere.
(2) Bob: Yes.

Next, the students placed a polaroid filter, oriented at 90° to the horizontal direction, in one of the arms of the interferometer, as shown in figure 7.

It was possible for the students to identify the polarization direction of the laser beam emitted by the source, as shown in the next dialogue.

Dialogue 2

(1) Alice: Here you can see that when the angle is zero, it is still on both of them.
(2) Bob: What?
(3) Alice: When the angle here is zero.
(4) Bob: Uh-huh.
(5) Alice: If it is zero, it means that it allows the laser to pass through both directions. When I turn it to 90° · · ·.
(6) Bob: It changes over here; have you seen it?
(7) Alice: Yes! So it blocks here where we have the polaroid. Which means · · ·.
(8) Bob: It does not let it go through.
(9) Alice: It does not let it go through. That's the polarization direction of the laser beam.

Right after that, the students removed the polaroid filter and replaced the second beam splitter, as shown in figure 8. The following dialogue shows that the students had no difficulty in interpreting the phenomena.

Dialogue 3

(1) Alice: It is the interference patterns.
(2) Bob: Yes.
(3) Alice: In the centre, one is constructive and the other is destructive.

Virtual interferometer operating in quantum mode

Selecting the single-photon option, the students removed the second beam splitter and replaced both screens by photon detectors, as shown in figure 9. They naturally interpreted the phenomenon in terms of probability, as shown in the following dialogue.

Dialogue 4

(1) Bob: It is random. 50% chance of transmission or reflection.
(2) Alice: Yes. It is random.

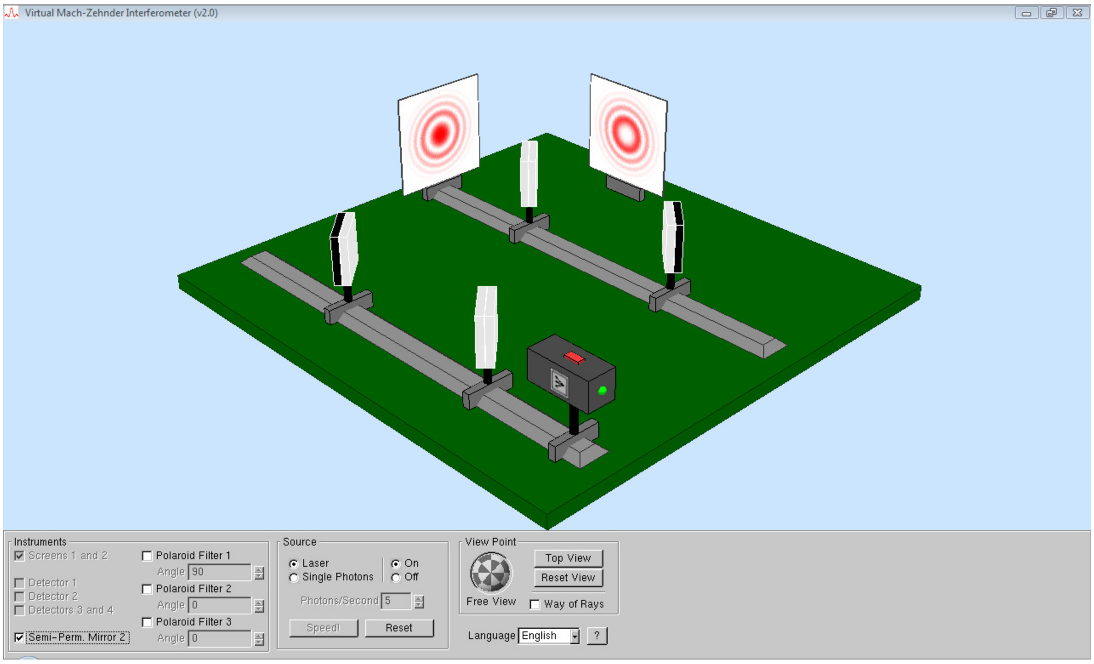

Figure 8. Ring pattern.

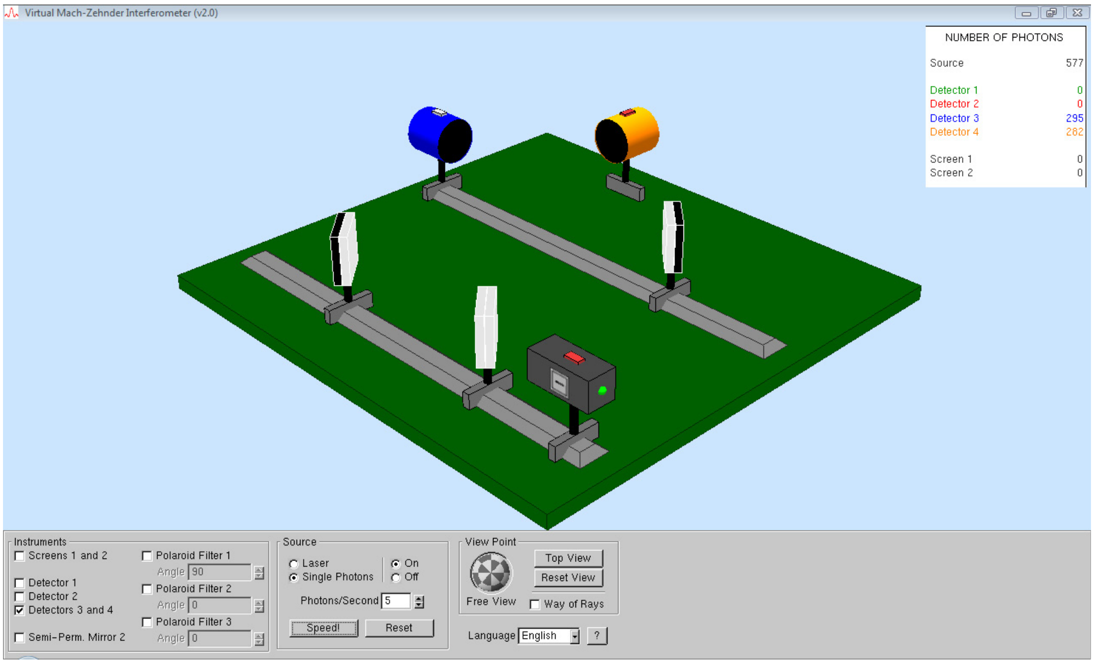

Figure 9. Photon indivisibility.

Next, the students replaced the second beam splitter, as in figure 10. This new set-up led the students into the following dialogue.

Dialogue 5

(1) Alice: Only one of them appears. Oh, of course! The interference pattern! If it comes

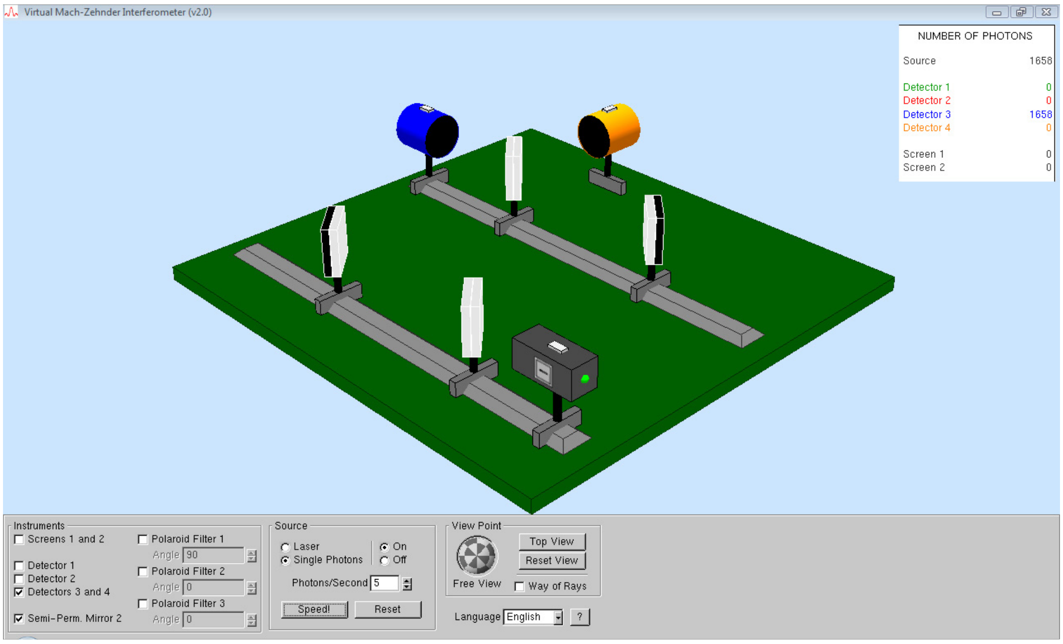

Figure 10. Quantum interference with detectors.

in the centre, is it not here where we had the · · ·.

(2) Bob: The constructive?

(3) Alice: Constructive · · ·. Where there was a bright centre · · ·. Where we got the photons · · ·. Then here I would really expect to have photons.

(4) Bob: It is just one photon at a time, right?

(5) Alice: One photon at a time.

(6) Bob: It does not divide itself!

(7) Alice: That is the point of quantum theory, isn't it?

(8) Bob: So if it is just one at a time, there could have been no difference, right?

(9) Alice: That is the point. It interferes with itself.

When the students again replaced the photon detectors by screens, as in figure 11, they obtained, for single photons, the same ring pattern as was previously observed (in dialogue 3).

Dialogue 6

(1) Alice: Now there is the beam splitter and there is interference. As you have a 50% chance (*of incidence*) here or here · · ·.

(2) Bob: Yes.

(3) Alice: But there is an interference pattern. You'll always have a bright centre.

(4) Bob: If it is one photon at a time, how does it interfere?

(5) Alice: Yes, if you imagine it as a corpuscle, it travels in one of the possible paths. But in quantum mechanics, it just · · ·.

(6) Bob:· · · is not valid.

While answering the question 'How would you explain the interference pattern observed for single photons?' the students reached an agreement, as stated below:

Dialogue 7

(1) Bob: We can say that classically one could not have any interference, while in quantum mechanics · · ·.

(2) Alice: In quantum mechanics, it looks as if the photon interacts with itself!

(3) Bob: But we must consider it as a wave.

(4) Alice: Yes.

Conclusions

In the literature, some authors propose the use of the Mach–Zehnder interferometer in order to discuss quantum interference phenomena as a starting point in the teaching of quantum physics

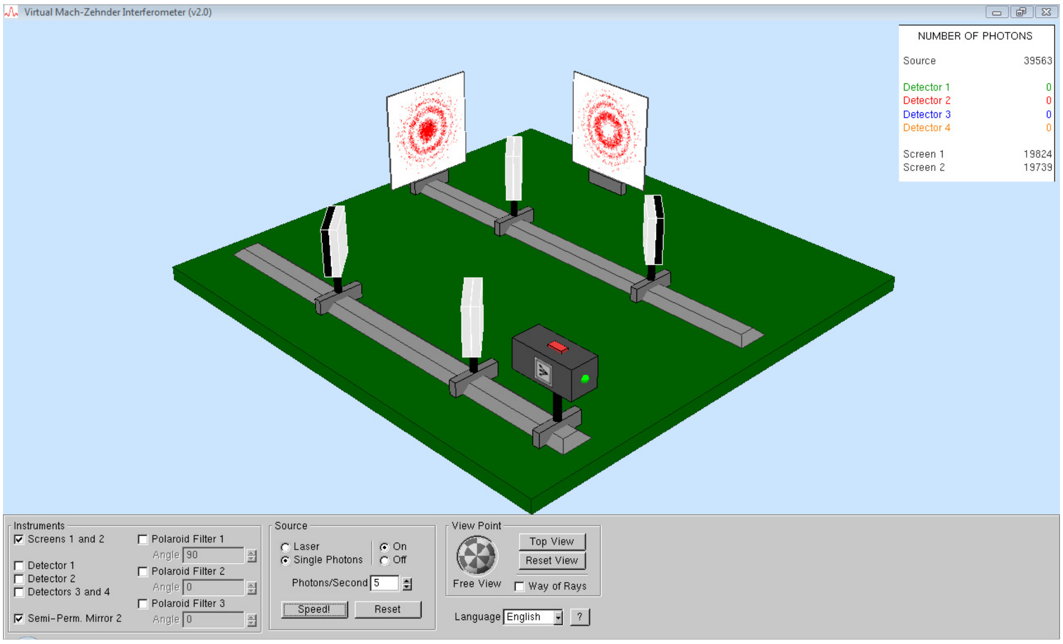

Figure 11. Quantum interference with screens.

(Pessoa Jr 2005, Scarani 2006). As a complement to this work we introduce the virtual Mach–Zehnder interferometer.

The sequence of activity suggested by the guide naturally led the students to establish an analogy between quantum physics and wave optics, as shown in the quotes from dialogues 5 (speeches 1 to 3) and 7 (speeches 3 and 4). Moreover, many of the phenomena observed in the computational simulation could show the photon's odd behaviour, avoiding the misconception in which quantum objects are seen as 'classical particles'. This can be observed in the quotes from dialogues 5 (speeches 4 to 9), 6 (speeches 4 to 6) and 7 (speeches 1 and 2).

These results suggest that this experience was a very important and positive factor in the education of these pre-service physics teachers. Through it, it was possible to contextualize a number of concepts and principles introduced in courses that were taken in previous semesters, such as those of photons, probability density, and quantum interference. In this way, the virtual Mach–Zehnder interferometer turned out to be a powerful tool, not only concerning the motivation for studying quantum physics, but also when it comes to improving the understanding of quantum phenomena.

The use of the Mach–Zehnder interferometer in a form of a computational simulation is justified by the lack of proper technological resources to carry out the same experiment using single photons in didactical laboratories. Such technology could only be achieved in the beginning of the 1980s in advanced physics laboratories. Recently, Galvez *et al* developed five experiments to demonstrate quantum interference for undergraduates (Galvez *et al* 2005), but the setup is too expensive for most schools. The virtual Mach–Zehnder interferometer, on the other hand, can be easily used in the high school physics classroom and it is available at the following Web address: www.if.ufrgs.br/∼fernanda/IMZ.

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Alexsandro Pereira received his undergraduate degree in physics in 2003. In 2008, he obtained his Masters degree in physics education from the Federal University of Rio Grande do Sul (UFRGS), Brazil. Currently, he is a PhD student in physics education at the same university. He also teaches physics to engineering students.

Fernanda Ostermann has a PhD in physics education from the Physics Institute of the Federal University of Rio Grande do Sul (UFRGS), Brazil. Currently, she works as an associate professor at this university, working mainly on the preparation of physics teachers and in graduate courses in physics education. She is also a productivity researcher of the National Council for Scientific and Technological Development (CNPq) in Brazil.

Cláudio Cavalcanti obtained his PhD in physics at the Physics Institute of Universidade Federal do Rio Grande do Sul (UFRGS), in plasma physics. Since 2006 he has been teaching and researching at UFRGS. His current research focus is on the teaching of modern physics at A-level and undergraduate levels.