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Does an Emphasis on the Concept of Quantum States Enhance Students' Understanding of Quantum Mechanics?

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Abstract. Teaching physics implies making choices. In the case of teaching quantum physics, besides an educational choice – the didactic strategy – another choice must be made, an epistemological one, concerning the interpretation of quantum theory itself. These two choices are closely connected. We have chosen a didactic strategy that privileges the phenomenological-conceptual approach, with emphasis upon quantum features of the systems, instead of searching for classical analogies. This choice has led us to present quantum theory associated with an orthodox, yet realistic, interpretation of the concept of quantum state, considered as the key concept of quantum theory, representing the physical reality of a system, independent of measurement processes. The results of the implementation of this strategy, with three groups of engineering students, showed that more than a half of them attained a reasonable understanding of the basics of quantum mechanics (QM) for this level. In addition, a high degree of satisfaction was attained with the classes as 80% of the students of the experimental groups claimed to have liked it and to be interested in learning more about QM.

1. Introduction[1]

Teaching physics implies making choices. In the case of teaching QM, besides an educational choice – the didactic strategy – another choice must be made, an epistemological one, concerning the interpretation of QM itself. Although these choices are not usually connected in the proposals that have recently appeared for the teaching of QM, we think that they are indeed closely connected. We have chosen a didactic strategy that privileges the phenomenological-conceptual approach, with emphasis upon quantum features of the systems, instead of searching for classical analogies. This choice has led us to present QM associated with an orthodox, yet realistic, interpretation of the concept of quantum state, considered as the key concept of QM, representing the physical reality of a system, independent of measurement processes.

Quantum theory is usually presented in university courses of Physics for scientists and engineers, and even in secondary school courses, with the help of approaches in which classical analogies and/or the historical evolution are highly valued. We have investigated the understanding of quantum concepts stimulated by those approaches, having as a theoretical framework Johnson-Laird's mental model theory, and we have found that these approaches do not allow a satisfactory understanding of most of quantum concepts, in consonance with the findings of several researchers regarding the conceptions of students about QM concepts (Niedderer 1987; Styer 1996; Johnston et al. 1998; Ambrose et al. 1999). Besides, these approaches exhibit the inconvenience of postponing students' contact with strictly quantum phenomena and seem to strengthen students' tendency towards creating classical analogies for quantum phenomena.

2. Quantum Mechanics Teaching

The success of QM, which can be evaluated by the variety of phenomena that it describes and predicts, as well as its impressive utilisation in modern technology, means that it is advisable that students preparing for many careers should undertake a study of QM as early as is possible. However, research into the teaching of QM is a recent topic in educational research (McDermott & Redish 1999), and important subjects such as student understanding of the superposition of states and the measurement problem have received scant attention (Greca & Moreira 2001). Researchers agree that the way in which QM is taught in introductory undergraduate courses is inefficient. They disagree about why it is inefficient, and about what can be done to improve the situation. The teaching of QM is often merely instrumental (technical), or simply a calculating course. Most courses seldom take the history of QM seriously. An exemplar of this can be found in Jones (1991, p. 93), who argues that the emphasis on ideas and pictures developed between 1900 and 1920 "produces wrong conceptual models, delaying understanding and interest". This approach does not please historians of science either. Kragh (1992), for instance, discussed in a comprehensive manner the history of the photoelectric effect as it is used in teaching this subject, concluding that is a case of quasi-history, with oversimplification and several mistakes.

Even if our analysis is circumscribed to QM introductory textbooks, it would be still worth including, both from historical and educational points of view, advanced textbooks in it, especially concerning about what interpretations have dominated them, once these books have been "training" physicists.

Despite the dominance of the Copenhagen interpretation between the wars – Jammer (1974, p. 250) wrote that this was the period of the "monocracy of the Copenhagen school" – this dominance was not expressed in textbooks. Kragh (1999, pp. 211–212) remarked that such a monocracy hid important nuances. This omission is important, as the textbooks were used to train professional physicists.

Kragh noted that Dirac (author of the most influential QM textbook), "did not see any point in all the talk about complementarity. It did not result in new equations and could not be used for the calculations that Dirac tended to identify with physics". Studying the way in which QM was received by American physicists, he affirmed that "American physicists had a more pragmatic and less philosophical attitude to physics than many of Bohr's associates", and concluded:

> That the contemporary importance of the complementarity principle was relatively modest is also seen from the textbooks from which students were taught quantum theory. Most textbook authors, even if sympathetic to Bohr's ideas, found it difficult to include and justify a section on complementarity. Among forty-three textbooks on quantum mechanics published between 1928 and 1937, forty included a treatment of the uncertainty principle; only eight of them mentioned the complementarity principle.

The absence of complementarity from textbooks worried several of Bohr's contemporaries, and some of them suggested that Bohr, himself, needed to write a book with a systematic and coherent presentation of complementarity.[2] As we know, that book was never written. His views need to be teased out of several epistemological papers. The relevance of this problem to the intellectual inheritance of Niels Bohr can hardly be underestimated. Abraham Pais (1991, p. 14), in his biography of Bohr, asks why complementarity "is not mentioned in some of the finest textbooks on physics, such as the one on QM by Paul Dirac, the historically oriented QM text by Sin-itiro Tomonaga, or the lectures by Richard Feynman?" He hoped his book could help to answer such a question.

In fact, all Bohr's discussion of complementarity as an epistemological innovation expressing limits to simultaneous use of magnitudes, the operators of which are non-commutable, is almost completely absent from textbooks. When some reference to it appears, it comes in the way of the mutual exclusiveness of particle and wave representations. Textbooks seem, as a matter of fact, to privilege what one could call an instrumentalist view of QM, or, according to Redhead (1987), what he named the 'minimal instrumentalist interpretation'; i.e., quantization algorithm, statistical algorithm plus the epistemological premise that "theories in physics are just devices for expressing regularities among observations". While reducing the cognitive reach of QM, and not making the understanding of QM easier, this dominance of the instrumentalist view and the absence of complementarity, seem to suggest that effectively physicists do not depend on complementarity in their professional praxis (Freire Jr. 1999, p. 212). Following this line, it is possible that the formalism of QM could itself carry an interpretation that has supported physicists in using and developing QM, even if this interpretation is not completely explicit or systematic (Paty 1992). It is this last possibility that has driven our choice about the interpretation one ought to privilege in introductory QM courses, a subject we will develop later.

3. The Mental Models Theoretical Framework

In order to investigate the kind of understanding developed by students exposed to traditional instrumentalist approaches to QM we have used Johnson-Laird's (1983) theory of mental models. According to it, to understand, explain or predict real or imaginary situations, we build in our minds working models, which are internal representations – idiosyncratic, functional, and incomplete – acting in an analog-structural way to things in the external world or in our imagination. In other words, facing a situation or phenomenon, both the elements chosen to interpret them, and the relations perceived or imagined among them, determine an internal representation, which functions as a substitute for that reality, from which it is possible to explain and predict the situation or phenomenon. Since states of things are many times described by concepts, the understanding of a concept also leads to the building of mental models. In this case, the nucleus of a mental model of a concept will represent its essence, i.e., the properties which are characteristic of the things the concept describes. Using this theoretical framework to study learning in physics courses, we can say students will understand physical models if they are able to build mental models adequate to those physical models. In that case they will be able to explain and predict phenomena described by those physical models in a way consistent with what is scientifically accepted.

However, most of the time such modeling does not occur. Students use fragmentary definitions and formulae, without understanding their meanings; and, according to the theory of mental models, the mental representations of these definitions and formulae are not referred to mental models and so quickly forgotten.

Studies on topics in classical physics subjects (Greca & Moreira 2002), have showed that frequently students' difficulties in understanding physical models are connected to the nuclei of concepts involved in building mental models to describe physical phenomena, such nuclei are also responsible for the kind of perception and reasoning about those phenomena. It seems that the essential characteristics of the concepts used by students to build mental models of physical models do not coincide with the essential characteristics of these concepts as included in accepted physical theories. From this point of view, in the transition from classical physics to quantum physics, students should give up nuclei that determine the 'classical' worldview.

Before continuing, it should be asked if one can speak of mental models in QM. Is it possible, and desirable, that students build structural analogues concerning quantum concepts? Are these concepts not too abstract to associate mental models to them? According to the theory of mental models, all concepts – abstract or not – require building mental models (Johnson-Laird 1983, p. 415). In other words, at the moment we come to understand how the world would be if such a concept was true, that means we have built mental models. Imagining non-Euclidean spaces is a similar question. Besides, we must take into account that in QM we are not speaking of visualizations that use mental images tightly connected to concrete objects. However, this is not an exclusive problem of QM, because many visualizations of classical physics are not true pictures anymore – see, for instance, possible visualizations of concepts such as field and energy.

4. How do Students Subjected to the Traditional Approach Visualize Quantum Concepts?

We have tried, therefore, to determine from what nuclei students understand phenomena and concepts related to the microscopic world. We have investigated two sections of Physics IV (Sophomore Engineering students) at the Federal University of Rio Grande do Sul, in Brazil. They had been exposed to the traditional approach to QM for one quarter of a semester. Besides, we have investigated one section of Quantum Mechanics I (Senior Physics) students who had been exposed to a technical approach. These students had been introduced, in previous disciplines, to concepts and mathematical skills necessary to follow that course. With instruments and technique we will present later, we have succeeded in identifying five of those nuclei concepts (Greca & Moreira 1999).

Particle – The main idea implied in this nucleus is that objects from the microscopic world are basically material particles, with some properties, in particular mass, and describe definite trajectories. Fifteen Engineering students seem to use this nucleus to build mental models of the concepts investigated. This idea supports mental models of electrons, photons, atom stability, and wave function, which is understood as the motion equation of a 'classical' particle.

Synthetic – Five Engineering students understand electrons and photons as having juxtaposed wave and particle properties. Electrons and photons are seen, some times, as being different manifestations of the same concept. The mental models built from this nucleus would have been used to explain electrons, photons, wave-particle duality (which is not a quantum duality since both coexist), wave function (as trajectories followed by particles) and photoelectric and Compton effects. This nucleus seems to be slightly different from the previous one. It permits students to include the duality idea into a particle matrix, and some students seem to use particle nucleus for some concepts and synthetic nucleus for others.

Blurred electron – Just for three Engineering students electrons can not be characterized as particles because they do not follow definite trajectories. From this, there is an uncertainty in their position, and the probability of finding them is given by wave function. Apparently photons do not share the same properties with electrons. This nucleus would be used to build mental models able to explain the stability of electrons in atoms, the uncertainty principle, wave functions, and probability distributions. However, these students do not correctly understand the uncertainty principle, as shown in their explanations and in the fact that, in any case, this principle appears related to the duality concept. Besides, this latter idea can not be explained from this nucleus.

Wavy – For two Physics students quantum phenomena are basically wavy ones. Material manifestation of particles is a consequence of their wavy properties. This nucleus would be used to build mental models of all quantum concepts and phenomena. These models, based on the conception of classical waves, do not result useful ideas for class – “This is my physics, not necessarily I work with it in my courses" (Student 2). Neither does it permit students to face the difficulty of "not understanding what they are doing".

Formal – For one Physics student quantum phenomena were understood from QM's formalism. He was the only one to attribute correct meanings, from the viewpoint shared by the physicists' community, to the investigated concepts.

The first four kinds of nuclei are variants of classical ways of visualization, and only the last, identified in one student of a most advanced course, could permit a right understanding of quantum phenomena and concepts. In the case of Engineering students, even if for some of them we could find evidences of formation of mental models to understand quantum phenomena and concepts (although the explanations and predictions that result from them were not adequate from an accepted scientific point of view), it seems evident that to most of them quantum concepts are fragmentary or mere mathematical expressions. That possibly occurs as a result of the difficulties in giving meanings to new information under 'classical' nuclei. For six other students of this group, it was almost impossible to identify how they had understood the concepts under investigation. In short, those students did not try to understand the new information; they restricted themselves to learning by heart.

The case of students from the advanced course is not different. Although those students seem to base their explanations on the wavy model, this model keeps classical elements, and they do not succeed in starting from this model to understand superposition or probability concepts. So they give priority to work with the formalism of QM. As to the others of this group, we have not identified how they 'visualize' any concept related to the microscopic world. For them, understanding is knowing an adequate algorithm to solve problems, without any preoccupation with their physical meanings. It's worth remarking that students from these groups have generally passed in their QM courses.

These results indicate that traditional approaches in introductory QM courses present scarce success: none of the concepts usually considered essential to describe quantum world seem to have been adequately understood by most of students. According to our theoretical framework, it is necessary to provide students with elements that will stimulate a 'perceptual change', i.e., replace the nuclei which are able to explain the 'classical world'. The traditional introductory courses approaches do not favor students learning the quantum way of perceiving phenomena; and latter courses, more technical ones, also do not seem to succeed in this goal either. On the contrary, they reinforce the procedures used by students – to decipher QM from classical codes previously learnt – by way of using classical and semiclassical models and analogies. Furthermore, the minimal instrumentalist interpretation which is behind these approaches does not leave much room for the understanding of quantum phenomena. In spite of the fact that physicists working with QM seem to develop, as result of their interaction with this theory, an interpretation that guides their scientific work, that is not the case of engineering students, whose formal contact with quantum theory is limited to these introductory courses. So such approaches have the further undesirable effect of preventing these students from getting in touch with strict quantum physics.

What would be these new perceptual relationships? What concepts should be implied in mental models adequated to the understanding of quantum phenomena? To answer these questions, we should analyze carefully the question of the interpretations of QM, since the approaches based on instrumentalist interpretation do not seem to succeed in terms of acquired learning of quantum concepts by students.

5. An 'Orthodox', but Realist, Interpretation of Quantum Theory

Our aim then was to help students to develop mental models whose results – predictions and explanations – coincide with that accepted by physicists' community. This led us to search a realist interpretation of QM because our remarks on scientific practice (Greca 2000) agree with Bunge (this issue) when he writes that "the realism [is] inherent in both common sense and the practice of science".

In order to arrive at such an interpretation, we have taken as our starting point the fact that several authors have suggested the possibility of an interpretation of quantum theory that, instead of changing the formalism, attempts to add new meanings to the usual terms of the theory. The key to such an interpretation consists in considering quantum states (represented by wave functions, state vectors) as having a physical reality independent of measurements.

Bohm and Hiley (1988) attributed this view to von Neumann, while opposing it to Bohr's view, because the latter valued in an excessive way the role of measurement, through the idea of wholeness of the system and the measurement apparatus. Bohm and Hiley, who were interested in showing that their interpretation in terms of a 'quantum potential' leads to a realistic view on EPR experiments, specially on its nonlocality, have emphasized a distinction, which was not always valued even in the specialized literature on the philosophy of QM. They have remarked that most physicists "do not follow the Bohr interpretation consistently", and that most of them "tend to use the von Neumann interpretation in terms of quantum, as represented by the wave function". The problem, according to them, is that "von Neumann's approach and Bohr's approach are different in certain key ways". In Bohr's description, "there is no room for any element of 'quantum reality' such as would be implied by the term quantum state". As von Neumman, differently, "assumes that the quantum state is the most complete description of reality that is possible", Bohm and Hiley conclude by suggesting that "indeed it would be fair to say that for von Neumann the quantum state is the basic element of reality".

It might be questionable whether one should attribute this view to von Neumann, because there is some incoherence between the realistic perspective of this view and the solution to the measurement problem he developed. As we know, it was this solution that has opened the road to the most subjective interpretations, such as those of London and Bauer, and Wigner, which attribute to the observer's mind the role for the transition, during a process of measurement, from a superposition of eigenvectors to the state with only one eigenvector.

As a matter of fact several physicists and philosophers – such as Fock, Bunge, Lévy-Leblond, and Paty – have suggested similar ideas, even though there are some relevant differences among them.

The Soviet physicist V. A. Fock, has played a role, in the controversy about the interpretation of QM, by his efforts aimed to dismiss any incompatibility between complementarity and dialectical materialism. He did this in the USSR during the period from 1947 to 1957, in which time incompatibility seemed be an official position of the Soviet Communist Party (Freire Jr. 1997). During his attempts, however, Fock was led to attribute more realism to complementarity than Bohr would perhaps be disposed to accept, criticizing some of Bohr's terminology, because it "gives rise to misunderstandings and to a positivistic interpretation of his ideas". In this way, the Soviet physicist emphasized that state vector in QM is an objective property of quantum systems, and describes the "potentially possible", while in classical physics the "potentially possible" is identified with the "accomplished". Then, to Fock, the quantum state describes real properties concerning real objects – atoms and molecules – independent of any measurement process.[3] He was inclined to see Bohrian complementarity as a "new element of relativity", which led him to introduce the idea of "relativity with respect to observation means" (Fock 1957).

Mario Bunge, after working for some time in developing hidden variable models suggested by David Bohm, has modified his perspective, while working on his project of an axiomatization of non-relativistic QM, a project the results of which were gathered in his book *Foundations of Physics*, published in 1967. At that moment, he realized that Bohm's was not a "valuable addition to standard QM, and that the solution to his (and de Broglie's and Einstein's) problems lay elsewhere, namely in a realistic reinterpretation of standard QM. It was not a question of injecting causality, but of ejecting the observer".[4] The difference between the realistic reinterpretation and the Copenhagen interpretation is subtle, though philosophically meaningful. As Bunge writes in this volume "instead of interpreting Born's postulate in terms of the probability of *finding* the quanton in question within the volume element Δv, the realist will say [...] that the probability in question is the likelihood of the quanton's *presence* in the given region". The Canadian-Argentinian philosopher was also among the first to use a new terminology – quantons – to describe QM as having an object, which did not include measurement process, essentially distinct from those ones of classical physics. Opening this *Science & Education* issue Bunge reaffirms his point of view, by emphasizing that quantum states have a physical reality, even if we can not attribute to them definite dynamical properties, because their values would be 'blurred'.[5]

Lévy-Leblond and Balibar (1990, p. 69), in a textbook with a innovative didactic approach to introductory QM courses akin to that we have developed, support similar epistemological premises. For instance, they use the same terminology suggested by Bunge – quantons – denying the idea that every physical object is either a wave or a particle. According to them, "it is, therefore, necessary to acknowledge that we have here a different kind of an entity, one that is specifically quantum. For this reason we name them *quantons*, even though this nomenclature is not yet universally adopted".

Michel Paty (1999a), more recently, has developed this idea "in terms of an extension of the meaning given to the concepts of *physical state and physical quantity* of a system, which would allow, without any theoretical change in QM, to speak consistently of *real quantum systems* as having definite *physical properties*". The philosophical key to this generalization was found by Paty (1999b) in a historical and epistemological analysis of the 'legitimacy of mathematization in physics'; this generalization suggesting "an extension of meaning for the concept of physical magnitude that puts emphasis on its relational and structural aspects rather than restraining it to a simple 'numerically valued' conception". According to the French philosopher, such a generalization could be useful not only to QM but also to the case of dynamical systems and quantum gravity.

While essentially based on his philosophical analysis, Paty argues with some questions related more directly to the scientific practice. He quotes the recent experimental confirmations of QM to maintain that the working physicist, in a spontaneous way, refer to quantum theory as "a fundamental theory about a given *world of objects*", and that this spontaneous perception only faces difficulties when focuses the "*transition* from this *quantum domain to the classical one*, that of measuring apparatuses". Paty remarks that such a suggestion of meaning generalization "has been guessed quasi explicitly by quantum theoretical physicists that were turned towards the formal properties of the theory, such as Max Born, Werner Heisenberg, Paul Adrien Dirac, John von Neumann and others", but "these pioneers did not however feel themselves authorized to propose from the start these formal constructions as directly conceivable as physical magnitudes, through a simple extension of meaning, because of the interpretation questions raised by then".

6. Central Concepts to be emphasized in QM's Teaching

The epistemological choice of a realistic interpretation, coupled with the adopted didactic strategy which will be explained later, led us to select an ensemble of central concepts to be emphasized in QM introductory courses. They should include concepts such as: *state superposition* (not only as a mathematical tool but also as a possible physical realization of a system); *Uncertainty principle* (as a limit to the use of canonically conjugated variables, and as an intrinsic property of quantum world, not as a limit of the measurement apparatus or ignorance of the system variables); *wave-particle duality* (quantum systems have sometimes properties that show some semblance of one or the other, but they are not identifiable either with wave or with particle); *distribution of probabilities* (which differ from the classical ones because they are irreducible and they interfere with one another), this distribution is well expressed in measurement problem – once measurement results of a magnitude upon a system in a pure state will be, with certain probability, one of its auto value, and after that the measurement system will be maintained in a state associated with the measured value; and finally, *nonlocality* (according to which acting upon a part of a system changes the state of the whole system, changing this way, 'instantaneously', the quantum description of the other part – spatially separated – of the same system).

7. The Didactic Strategy

As one of the main problems presented by students in QM courses relates to the difficulty in creating an adequate perception for the microscopic world, abandoning classical forms of perceiving the phenomena, we ask ourselves how to help them to acquire a new perception. We propose in this context a phenomenological-conceptual approach, focussing on experiments and observations that emphasize the very first principles of QM. In consonance with the realistic interpretation we have adopted, the fundamental goal of this procedure is to make first principles become "palpable", not just mathematical relations with far or doubtful connections to the physical world. We believe that presenting direct consequences of such principles on reality will help the students to visualize the quantum world and, in consequence, to construct adequate mental models to describe it. In order to reach this objective, we developed an instructional unit (24 hours, during one month), characterized by a spiral structure where the most important concepts reappear at different examples. Tutorials were complemented by partial explanations by the teachers and small groups' discussions with teachers' assistance. The main concepts discussed in the unit were the previously quoted, that is, the superposition principle, the uncertainty principle (wave-particle duality), probability distribution and the measurement problem. Classical concepts and models have been avoided, and the twelve lectures that composed the unit included phenomenological examples always centered on the manifestations of the quoted principles. Besides, when discussing with students in class, references that could be interpreted as emphasis on wave or particle models were avoided. On the contrary, the idea of considering quantum systems as 'quantum objects' (this was the terminology used) was emphasized, showing the different behavior of these objects when compared with those of the macroscopic world. Whenever possible, one spoke of quantum objects in a general way, without discriminating the distinct objects. In the same search for generalization, other physical magnitudes besides energy were introduced.

These tutorials included a list of conceptual questions and short problems with which students should work in classrooms. Whenever possible, the questions were not placed at the end of each text, but introduced in the middle. These questions and problems aimed to complete the reasoning exposed in the text, therefore the texts were not prepared to be read in a passive way by students. As an instance of our strategy, experiments that had been directly analyzed for the superposition of states included the Stern-Gerlach (spin) and Young (one particle self-interference) experiments, the Schrödinger's cat paradox, quantum computing and teleportation, also introducing some ideas about the decoherence theory. The mathematical demands were minimal and in the presentation of solutions of the Schrödinger equation, which calls for some knowledge about differential equations, computer software were used to help students to visualize and vary the required solutions. The distribution of students in small groups for discussion among themselves and with the teachers was very important: in order to understand the texts and to answer the questions included in them, students were urged to express their personal form of seeing the different phenomena, externalising contradictions and difficulties. This process allowed the students to revise and improve their mental models, and it is based on Vygotsky's theory of mediation (Szorzio 1995).

The project here described was implemented in three separate terms of General Physics in the fourth semester of Engineering courses, at the Universidade Federal do Rio Grande do Sul, Brazil, during the first and the second semesters of 1999.

8. Methodology and Data Treatment

With regard to our proposal, we tried to find out if the conceptual nuclei developed with this approach are more suitable than the ones built with the traditional approach as far as the correct understanding of basic quantum concepts is concerned. We present here the results obtained with two of the experimental groups (N = 69), named A and B. These results are compared with those of two others groups, one constituted of students of the same general physics class, but developed under the traditional approach (N = 10), named C, and the other constituted by physics students enrolled in an introductory course of QM under the traditional approach and with a duration of 90 hours (N = 10), named D.

8.1. DATA TREATMENT

We have adopted the following assumption about the nuclei of the mental models: if the nuclei determine how the different concepts and phenomena are perceived, then they also determine if certain concepts and phenomena are perceived as similar or not. So, the knowledge of the way students associate certain concepts may give clues about the mental models used to understand these concepts (Kearney & Kaplan 1997). Data were gathered from concept association tests and conceptual problems. On the whole 15 concepts and 3 problems were used in pre and posttests. Concepts presented to the students referred to: the state of a physical system, observables, tunnel effect, linear superposition of states, probability distribution, eigenvalues, quantum particle trajectory, electron, photoelectric effect, wave function, measurement results, wave-particle dualism, simultaneous observables, expectation values and uncertainty principle; and the problems involved the major concepts discussed. Furthermore the students were asked to explain in the posttest, with their own words, what they had understood about each one of the concepts included in the test.

The data obtained from the questions and the written explanations were analyzed qualitatively in order to characterize the nuclei of the mental models used by the students. Additionally, in the case of the experimental groups, field notes collected during the classes were also considered. The data from the association tests were transformed into relatedness coefficients – considered as a measure of relationship between concepts – calculated according to Garskov & Houston (1967) for each pair of concepts and for each student. These coefficients provided individual matrices of similarity from where we calculated average similarity matrices, which were analyzed by using the hierarchical clustering and the multidimensional scaling techniques. For each one of the categories of students considered above and for the groups taken for comparison, average matrices were calculated and analyzed by using the INDSCAL procedure (Borg & Groenen 1997), a special form of multidimensional scaling that permits researchers to account for individual differences in the perceptual processes that generate student responses. Thus we used an integrative methodology that combined qualitative and quantitative methods.

9. Results[7]

Four categories emerged from the qualitative analysis.

Category 1: Quantum object nucleus (25% of the students from the experimental groups) – The students of this category seem to have incorporated in their mental models the principal concepts introduced to describe the microscopic world: uncertainty principle (duality), probability distribution, and superposition of states. They succeeded in answering the proposed questions and gave good explanations, considering their level of instruction, for the concepts included in the tests. These students described quantum phenomena from general principles and established distinctions between classic and quantum concepts. They also presented adequate relations for the states of the system before and after measurement.

Category 2: Incipient quantum object nucleus (40% of the students from the experimental groups) – These students seem to have incorporated some of the main concepts – uncertainty principle (duality) and probabilities. They were able to answer and to explain questions in which these concepts were involved, but showed difficulties in understanding the linear superposition of states. This was evident in that, in opposition to the students of the previous category, none of them succeeded in presenting clear explanations about the expected information on states of a system before and after measurements. In general, they quoted several examples used during the teaching to reinforce their explanations.

Category 3: Classical nucleus with some quantum ingredients (18% of the students from the experimental groups) – This category seems to use classical nuclei to visualize quantum phenomena. This causes a classical distorted incorporation of the concepts already mentioned. Typical examples of this distorted vision are:

considering the electron as a classical particle with a defined trajectory (in some cases, the trajectory is identified with the wave function), the superposition of states as being a superposition of the wave and particle behaviors of an object, and the uncertainty principle as resulting from the ignorance about the variables in the problem or from inaccuracies of the measurement process. Such students either did not answer the proposed questions or presented classical answers. Although their answers were wrong, they seemed to try to "understand" the subject, with the material discussed in classroom making sense to them. These students seemed to use the nuclei we have named particle and synthetic in Section 4.

Category 4 : Undetermined (17% of the students from the experimental groups) – For this last category of students it was not possible to find any pattern in the responses. Their explanations showed no nexus: in most cases they did not even answer the questions or, very often, they just repeated definitions presented during the classes.

As to the students from the groups receiving instruction in the traditional approach, they were basically located in the last two categories. This was the same result as in our earlier study. None of them seemed to have incorporated fundamental quantum concepts in order to construct useful mental models to understand the microscopic world: they did not have any explanations at all, or, if they did, those were consequences of classical kinds of interpretation of microscopic phenomena. They also got pass rates significantly lower than those ones the students of experimental groups did.

The main result for the INDSCAL procedure appears in Figure 1, which shows two dimensions of the subject space configuration – the space where categories appear – analyzed in five dimensions. This accounted for 53% of the variance in all data of subjects emerging from the INDSCAL procedure.

The dimensions plotted in the figure are the ones that better divide the groups. The correct interpretation of the graph is that the points plotted for each group represent the extremity of vectors drawn from the origin of the space, the most important characteristics being the direction of these vectors (Wish et al. 1972). With this in mind, we can observe that it is possible to draw a line from the origin, so that all the groups below the line are groups that do not form quantum nuclei. The groups above the line are the ones for which it was detected that they form quantum nuclei. For the groups below the line, dimension 5 is more important, while dimension 4 is more important for the groups above it. When looking at what these dimensions stand for in the stimulus space – where the concepts presented to students appear, and which are not shown here – it can be seen that dimension 4 established both the classification and hierarchy between the concepts considered. This structure disappears when we look at dimension 5 alone, which shows that the subjects that found this dimension more important have a worse understanding in comparison to others (Greca, 2000). These results seem to confirm the proposed categorization.

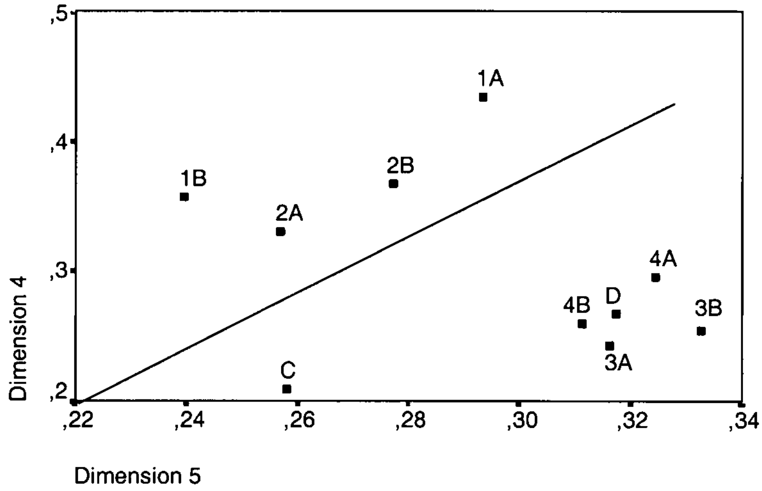

Figure 1. Capital letters stand for the different groups: A and B, experimental groups, C and D, comparison groups. The numbers represent the different categories.

10. Conclusion

The results of the analysis performed with respect to the implemented project show that the short introductory course grounded on a realistic interpretation of the basic concepts of QM gave rather favorable learning responses. A total of 65% of the experimental groups showed a reasonable understanding of the basics of QM for this level. In addition, a high degree of student satisfaction was attained, as 80% of the students of the experimental groups declared they liked the course and that they were interested in learning more about QM.

We consider that our work also shows another way in which philosophy of science can be used to develop didactic strategies to learn scientific concepts. In general, contributions of philosophy to the teaching of science come from two main sources: criticism of the usual idea of scientific method, and the use of ideas concerning the development of science (by Popper, Kuhn, Lakatos, Bachelard, and so on) to a better understanding of the process of acquisition of scientific concepts by students, in order to elaborate didactic strategies in agreement with them (Matthews 1994, pp. 83–108). In our case, we have used elements of philosophy of a specific scientific theory to guide the choice of the didactic strategy to be followed.

On the inverse direction, it seems to us that a method of studying the didactic consequences of different interpretations of QM, and searching for the conceptions students acquire from them, could be an interesting subject to be considered in debates about different QM interpretations.

Therefore we would like to conclude by paraphrasing Bunge's morals (this issue). *Moral 1*: The science teachers who do not take into account the philosophical premises of the subject being taught risk not enhancing their students' understanding. *Moral 2*: The philosophers of science who do not take into account the educational implications of their philosophies risk not to explore the full meaning of their theories.

Notes

[1] This paper presents results of a doctoral thesis by one of the authors (Ileana Greca). Its title is "Construindo significados em mecânica quântica: resultados de uma proposta didática aplicada a estudantes de física geral", (Building Meanings in QM: Results from a Didactic Proposal Applied to Physics Course Students), Physics Institute – UFRGS, Porto Alegre, Brazil, 2000. This thesis was advised by Dr. Marco Antonio Moreira, co-advised by Dr. Victoria E. Herscovitz, and supported by CNPq. Hence, the joint contribution of both authors of this paper is restricted to Sections 2 and 5.

[2] Philipp Frank wrote to Bohr, on 4 June 1952: "I have found that you have formulated the answers to these problems in a much more lucid way than most authors. I have found on the other hand that the current textbooks which treat these questions don't present your views in a very adequate way [...]. On the other hand, you have never presented your views in a comprehensive book but only in single papers". In a letter (4 March 1960), not sent, from Bohr to David Park, the former writes: "For many years we have ourselves felt this want [the lack of up-to-date, orderly presentation of complementarity], and have planned a comprehensive account of the foundations of quantum theory, which we hope to complete in the near future". *Archive for the History of Quantum Physics*.

[3] Fock (1957, pp. 643–651) argued that: "If a subdivision of the experimental arrangement in a preparatory part, a working part and a registration part is possible, one can vary the last stage of experiment and obtain probability distributions referring to the same initial state. Since these are parametrically expressed in terms of one and the same wave function, this function is independent of the last stage of experiment". This independence "on the final stage allows one to make an abstraction on it and to consider the wave function as an objective characteristic of the state of the object just before the final stage".

[4] Mario Bunge, Letter to O. Freire Jr. (1–11–1996).

[5] In this volume (p. 5) Bunge writes on the meaning of vectors in QM: "The quantal arrows are so blurred in both breadth and direction, that they do not look at all like arrows". Before finishing commenting on Bunge's interpretation we must say we don't agree with him at length. For instance, he writes (p. 10), "Bohm's attempt eventually met with defeat in the laboratory". The source of this mistake seems to rest on where he identifies (p. 16) "local hidden variable theories" with " the whole family of hidden variable theories", because Bohm's theory was not a local one.

[7] We present here a resume of the results. For a larger presentation and discussion, see Greca & Herscovitz (2001), Greca et al. (2001).

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